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Renegotiation Policies in Sovereign Defaults*

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ABSTRACT

This paper studies an optimal renegotiation protocol designed by a benevolent planner when two countries renegotiate with the same lender. The solution calls for recoveries that induce each country to default or repay, trading off the deadweight costs and the redistribution benefits of default independently of the other country. This outcome contrasts with a decentralized bargaining solution where default in one country increases the likelihood of default in the second country because recoveries are lower when both countries renegotiate. The paper suggests that policies geared at designing renegotiation processes that treat countries in isolation can prevent contagion of debt crises.

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The current European debt crisis has prompted intense discussions among policy makers about potential policies to curtail crises. Some countries, in fact, have established new institutions or strengthened existing ones, with a mandate for financial stability.[1] A growing academic literature on macroprudential policies suggests that policies that reduce borrowing can be welfare improving in environments of pecuniary externalities or sticky prices.[2] Nevertheless, in the data, debt crises tend to happen in multiple countries at the same time, and this literature is silent about policies that prevent contagion. Arellano and Bai (2013) show that a powerful mechanism for the bunching of defaults is the strategic complementarities among countries in renegotiation procedures. In this paper, we show that policies designed to improve renegotiation procedures can prevent contagion in debt crises.

This paper studies an optimal renegotiation protocol designed by a benevolent planner when two countries renegotiate with the same lender. The solution calls for recoveries that induce each country to default or repay, trading off the deadweight costs and the redistribution benefits of default independently of the other country. This outcome contrasts with a decentralized bargaining solution where default in one country increases the likelihood of default in the second country because recoveries are lower when both countries renegotiate. The paper suggests that policies geared at designing renegotiation processes that treat countries in isolation can prevent contagion of debt crises.

The model consists of two borrowing countries that decide to repay or default on the debt they owe to a lender. All agents have linear payoffs. Default has real costs as it reduces the output of the country. After default, countries renegotiate the recoveries with the lender. Failure in renegotiation leads to a further drop in countries' output. Countries decide to default if the resources from defaulting and paying the recovery are greater than the resources from their income net of paying the debt.

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[1]As described in Angelini et al. (2012), newly established macroprudential institutions are the European Systemic Risk Board (ESRB) in the European Union and the Financial Stability Oversight Council (FSOC) in the United States. In Britain the 2009 Banking Act gives the Bank of England broad powers in this field.

[2] See, for example, Bianchi (2011), Bianchi and Mendoza (2013), Lorenzoni (2008), and Farhi and Werning (2013).

In the decentralized renegotiation process, we consider a generalized Nash bargaining protocol where all the renegotiating countries simultaneously bargain over recoveries of their debts. As is standard, the solution of the protocol assigns each player a fraction of the surplus from the renegotiation, which equals countries' output in default relative to countries' output in case of renegotiation failure. The recovery that each country pays is lower when all three players bargain because the effective weight of the lender is lower in this case. These patterns of recoveries imply that for intermediate levels of debt, a default in one country leads to a default in the other country. This dependency arises during fundamental defaults abroad, where the other country defaults because of high debt, and also during self-fulfilling defaults, where both countries default only because the other is defaulting.

In the planning problem, a benevolent planner decides on recoveries that maximize the weighted sum of each player's consumption. As in the decentralized problem, the planner needs to satisfy the participation constraints of the borrowing countries and the lender, which requires that the surplus from the renegotiation is positive for all players. The planner chooses recoveries to generate the desired default and repayment patterns as well as the allocations of consumption during default. Default is costly because it destroys resources, but it has redistribution benefits when the planner assigns a weight to each borrower that is larger than that assigned to the lender. By setting a sufficiently low recovery, default is the only way for the planner to reduce the transfers from the borrowers to the lender .

Default is optimal for the planner when the redistribution benefits from default are higher than its deadweight costs. The planner can achieve the maximum redistribution benefit by setting the recovery to the lower bound. The lower bounds of recoveries are determined by the lender's participation constraint, which requires that the sum of the recoveries from both countries is greater than or equal to zero. This implies that in the planning problem, both countries are linked precisely because of this lower bound of recoveries. When only one country is renegotiating, the lower bound of recovery is equal to zero. When two countries are renegotiating, the lower bound of recovery for one country is equal to the negative of the recovery of the other country.

We show that for a given recovery, the planner has lower default incentives than each country because the redistribution costs for the lender from default factor into the planner's problem, whereas this factor is irrelevant in the decentralized model. The main result of the paper, however, is that in the planning problem, recoveries for each country are chosen independently of the other country. For low levels of debt, the planner sets a high enough recovery such that it induces repayment. For high levels of debt, the planner prefers default and sets recoveries equal to zero. Default patterns in each country are independent of those of the other country because any potential benefits from cross-subsidizing one borrowing country from the other borrowing country have no value for the planner. Both borrowing countries are weighted equally for the planner, and inducing another default adds additional resource costs.

The takeaway from our analysis is that the strategic complementarities that arise from a decentralized bargaining process are welfare reducing because they are eliminated in a planning problem. Arellano and Bai (2013) show that such strategic complementarities, which arise from lower recoveries when multiple countries renegotiate, are a powerful and empirically relevant contagion mechanism for sovereign debt crises. The analysis of this paper, therefore, suggests that an important role for policy aimed at reducing contagion in debt crises is the design of renegotiation protocols that treat each country in complete isolation.

I. Renegotiation Models

We now present two models of debt renegotiation between a lender and multiple borrowers. We compare the outcome of a noncooperative decentralized bargaining model with the outcome of a centralized planning model.

The economy consists of two borrowing countries, $i = 1, 2$, and one lender. The two countries start with some income y_i and debt b_i owed to the lender. The lender has a constant endowment y_L. Countries decide whether to default on the debt, $d_i = 1$, or repay it, $d_i = 0$. Default entails costs in that income falls to $y_i^d \leq y_i$. Defaulting countries can renegotiate with lenders by paying the recovery ϕ_i. Failure to renegotiate further reduces the income to $y_i^{nr} \leq y_i^d$. All agents have linear payoffs.

A. Decentralized model

The decentralized model we consider is a one-period version of the renegotiation model in Arellano and Bai (2013). During renegotiation, countries bargain over the recovery ϕ_i, the lender with Nash bargaining. The renegotiation protocol links the two recoveries, which matter for the countries' default decisions. Formally, country i decides whether to default to maximize its payoff: $x^d(d_{-i}) = \{d_i : \max_{d_i=\{0,1\}}(1-d_i)(y_i - b_i) + d_i(y_i^d - \phi_i(d_{-i}))\}$. $x^d(d_{-i})$ denotes the best response default decision of country i given country $-i$'s default decision d_{-i}. Countries' default decisions and recoveries determine the lender's payoff:

$$(1-d_1)(1-d_2)(y_L + b_1 + b_2) + \sum_{i,j} d_i(1-d_j)(y_L + b_i + \phi_j) + d_1 d_2 (y_L + \phi_1 + \phi_2).$$

The recovery value ϕ_i when only country i renegotiates satisfies a standard Nash bargaining problem: $\max_{\phi_i}\{(y_i^d - \phi_i) - y_i^{nr}\}^{\lambda_B}\{y_L + b_{-i} + \phi_i - (y_L + b_{-i})\}^{\lambda_L}$, subject to the participation constraints for both the borrower $(y_i^d - \phi_i) - y_i^{nr} \geq 0$ and the lender $y_L + b_{-i} + \phi_i - (y_L + b_{-i}) \geq 0$. The terms λ_B and λ_L are the bargaining weights of the countries and the lender, respectively.

The outcome of the above problem is such that the recovery equates the marginal cost of country i with the marginal benefit of lenders and satisfies

$$\phi_i^S = \frac{\lambda_L}{\lambda_B + \lambda_L}\left(y_i^d - y_i^{nr}\right). \tag{1}$$

The solution is intuitive. The total surplus from renegotiation is $y_i^d - y_i^{nr}$. The fraction of surplus that lenders receive depends on their bargaining power relative to that of borrowing countries, $\lambda_L/(\lambda_B+\lambda_L)$.

The recovery values ϕ_1 and ϕ_2 when two countries renegotiate satisfy the following generalized Nash bargaining problem: $\max_{\phi_1,\phi_2}\{(y_1^d-\phi_1)-y_1^{nr}\}^{\lambda_B}\{(y_2^d-\phi_2)-y_2^{nr}\}^{\lambda_B}\{(y_L+\phi_1+\phi_2)-y_L\}^{\lambda_L}$, subject to the same participation constraints for the two borrowers and the lender, which is $(y_L+\phi_1+\phi_2)-y_L\geq 0$.

The solution to this problem implies that the recoveries for countries $i=1,2$ satisfy

$$\phi_i=\frac{\lambda_L}{\lambda_B+\lambda_L}\left(y_i^d-y_i^{nr}\right)-\frac{\lambda_B}{\lambda_B+\lambda_L}\phi_{-i}. \tag{2}$$

The expressions (1) and (2) imply that when two countries renegotiate together, the recovery for country i is lower than when it renegotiates alone if the other country is paying a positive recovery $\phi_{-i}\geq 0$. Furthermore, the two recoveries from (2) can be written as

$$\phi_i^J=\frac{\lambda_L}{2\lambda_B+\lambda_L}\left(y_i^d-y_i^{nr}\right)-\frac{\lambda_B}{2\lambda_B+\lambda_L}\left[\left(y_{-i}^d-y_{-i}^{nr}\right)-\left(y_i^d-y_i^{nr}\right)\right]. \tag{3}$$

If the surpluses from renegotiation are equal among countries, then recoveries are lower for both countries when the countries renegotiate together.[3] If the surplus for country $-i$ is larger than that for country i, there is cross-subsidization across countries during renegotiation; the recovery for country i decreases as its surplus relative to the other country falls.

Given these recoveries, the default best-response decisions are cutoff rules: $x_i^d(d_{-i}=0)=1$ if $b_i>B_i^S$, $x_i^d(d_{-i}=1)=1$ if $b_i>B_i^J$, otherwise the country repays $x_i^d=0$. The debt cutoffs above which the country defaults are $B_i^S=y_i-y_i^d+\lambda_L\left(y_i^d-y_i^{nr}\right)/(\lambda_B+\lambda_L)$ and $B_i^J=y_i-y_i^d+\{(\lambda_L+\lambda_B)\left(y_i^d-y_i^{nr}\right)-\lambda_B\left(y_{-i}^d-y_{-i}^{nr}\right)\}/\{2\lambda_B+\lambda_L\}$. We assume that the

[3]Arellano and Bai (2013) show that empirically, recoveries are lower in years when many countries renegotiate. They use the historical dataset of debt recoveries by Cruces and Trebesch (2013) and find that recoveries are 16 percentage points lower in years during which four or more countries finish their renegotiations.

surpluses of both countries are not too different, which implies that $B_i^J < B_i^S$ for $i = 1, 2$.[4]

B. Planning problem

Now we consider the planning problem where a benevolent planner sets and commits to the recoveries for the two countries $\{\phi_1^P, \phi_2^P\}$ before default decisions. Given the announced recovery ϕ_i^P for country i, the country chooses to default or not to maximize its consumption: $\max_{d_i}(1 - d_i)(y_i - b_i) + d_i(y_i^d - \phi_i^P)$.

The planner chooses recoveries $\{\phi_1^P, \phi_2^P\}$ to maximize the weighted sum of utilities for the two borrowing countries and lenders.The term λ_B is the planner weight on the borrowing countries, and λ_L is the weight on lenders. We assume that $\lambda_B > \lambda_L$. The planner internalizes that the recoveries determine default outcomes. As in the decentralized model, the planner has to respect the participation constraint for each agent. The planning problem is

$$(4) \qquad \max_{\phi_1^P, \phi_2^P} \{\lambda_B(y_1 - b_1) + \lambda_B(y_2 - b_2) + \lambda_L(y_3 + b_1)$$
$$+ I_{y_1 - y^d < b_1 - \phi_1^P}[\lambda_B(y_1^d - y_1) + (\lambda_B - \lambda_L)(b_1 - \phi_1^P)]$$
$$+ I_{y_2 - y_2^d < b_2 - \phi_2^P}[\lambda_B(y_2^d - y_2) + (\lambda_B - \lambda_L)(b_2 - \phi_2^P)]\}$$

subject to the bounds on recoveries that arise from the agents' participation constraints. If only country i defaults, that is, $y_i - y_i^d < b_i - \phi_i^P$ and $y_{-i} - y_{-i}^d \geq b_{-i} - \phi_{-i}^P$, then $\phi_i^P \in [0, y_i^d - y_i^{nr}]$. If both default, $y_i - y_i^d < b_i - \phi_i^P$ for $i = 1, 2$, then $\phi_i^P \in [-\phi_{-i}^P, y_i^d - y_i^{nr}]$ for $i = 1, 2$.

The planning problem contains interactions between default decisions of countries only because the feasible set for recovery varies with single or multiple defaults. When only one country defaults, the recovery needs to be nonnegative $\phi_i^P \geq 0$ to satisfy the lender's participation constraint. When both countries default, then the lender's participation constraint implies that the recovery has to be at least as large as the negative of the recovery for the other country, $\phi_i^P \geq -\phi_{-i}^P$.

[4] The parameter restrictions are $\lambda_B/(\lambda_B + \lambda_L) < (y_i^d - y_i^{nr})/(y_{-i}^d - y_{-i}^{nr}) < (\lambda_B + \lambda_L)/\lambda_B$.

We require that the participation constraints for agents are also satisfied in off-equilibrium events. If repayment is optimal for both countries, $\phi_i^P \in [0, y_i^d - y_i^{nr}]$ for $i = 1, 2$. If repayment is optimal for i but default is optimal for $-i$, $\phi_i^P \in [-\phi_{-i}^P, y_i^d - y_i^{nr}]$.

Recoveries determine default repayment patterns and the consumption of the agents in case of default. The objective function (4) makes it clear that default is optimal for the planner in country i if and only if $[\lambda_B(y_i^d - y_i) + (\lambda_B - \lambda_L)(b_i - \phi_i^P)] \geq 0$. This expression says that default is optimal if the weighted deadweight costs of default $\lambda_B(y_i^d - y_i)$, a negative number, are less than the benefit of the default to the borrower relative to the cost for the lender, $(\lambda_B - \lambda_L)(b_i - \phi_i^P)$. Note that for default to ever be optimal, the borrower's weight has to be sufficiently larger than lender's.

Lemma 1. *Given recovery ϕ_i, country i prefers to default for lower levels of debt than the planner.*

The planner prefers to default for higher levels of debt because it also weights the redistribution losses of lenders from default. An immediate implication of this result is that the planner will set the recoveries ϕ_i^P to curtail default.

Lemma 2. *Let $\underline{\phi}_i$ be the lower bound of recovery for country i. The planner prefers default for country i if $\lambda_B(y_i^d - y_i) + (\lambda_B - \lambda_L)(b_i - \underline{\phi}_i) \geq 0$ and prefers repayment otherwise.*

The value of default to the planner is the greatest the lower the recovery ϕ_i^P, so the relevant debt threshold b_i to consider is the one that maximizes the additional value the planner gets from default. The lower bound on recovery equals zero when one country is defaulting and equal to the negative of the recovery of the other country when both countries are defaulting, $\underline{\phi}_i = \{0, -\phi_{-i}^P\}$. The planner hence controls these bounds by inducing one or two countries to default. The interdependence across countries in the lower bounds on recovery is what links the decision of the planner to default or repay across countries.

Lemma 3. *When anticipating a single default from country i, the planner sets $\phi_i^P = 0$. When anticipating joint defaults, the planner sets $\phi_1^P + \phi_2^P = 0$.*

When only one country defaults, the lower bound $\underline{\phi}_i = 0$ and the planner wants to induce default if $\lambda_B(y_i^d - y_i) + (\lambda_B - \lambda_L)(b_i - 0) \geq 0$. The planner might also be forced to have one country default when the country has such a high level of debt that default is preferred even under the highest recovery value $\bar{\phi}_i = y_i^d - y_i^{nr}$. In this case, the planner has no tools to prevent the default and sets $\phi_i^P = 0$ to minimize the losses from default even if $\lambda_B(y_i^d - y_i) + (\lambda_B - \lambda_L)(b_i - 0) \leq 0$. A similar logic applies when the planner anticipates a default in both countries. Setting the sum $\phi_1^P + \phi_2^P$ to zero maximizes the value of the planner from the default, independently of whether defaults are preferred or forced for the planner. The next proposition summarizes the cutoffs of debt above which default occurs for each country in the planning problem. The cutoffs are the minimum of those where the default increases the planners' objective or is forced to default.

Proposition 1. *Country i defaults in the planning problem iff $b_i \geq B_i^P$ where*

$$B_i^P \quad = \quad \min\left\{\frac{\lambda_B}{\lambda_B - \lambda_L}(y_i - y_i^d), \quad y_i - y_i^{nr}\right\} \textit{ for } i = 1, 2.$$

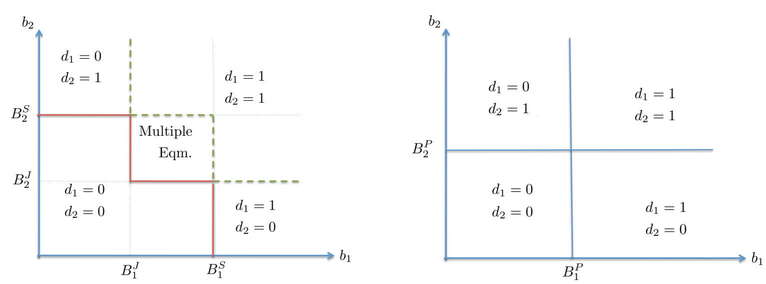
We relegate the proof to the appendix. This proposition says that the default outcome for each country i is independent of all states and outcomes of country $-i$. Manipulating the recovery of one country to induce the other country to default implies a cross-subsidization across countries. Cross-subsidizing is never optimal because the associated redistribution benefits are a wash for the planner who values both borrowing countries equally, and any additional default from such a policy induces additional costs.

In repayment states, when debt is lower than the cutoff $b_i \leq B_i^P$, the recovery the planner sets is high enough such that it prevents default. In general, there are many recovery values that deliver repayment and without loss can be assumed to be the upper bound $\bar{\phi}_i$. When $b_i > B_i^P$, then the planner sets the recovery to 0 and country i defaults.

C. Comparing the Decentralized and Planning Problems

We now compare the outcomes of the planning problem with the outcomes of the decentralized problem in terms of default sets and consumption.

A main difference between the two outcomes is that the planning problem eliminates the dependencies across the two countries in default. Panel (a) of Figure 1 illustrates the equilibrium default and repayment in the decentralized model. Panel (b) plots the equilibrium in the planning problem described in Proposition 1. In the decentralized model, for intermediate levels of debt, both countries default only if the other country defaults too. Such strategic complementarities lead not only to dependencies but also to self-fulfilling defaults. In contrast, in the planning problem, each country defaults if the level of debt is above a unique threshold. This result implies that eliminating the strategic complementarities in the decentralized model is welfare improving for all agents.



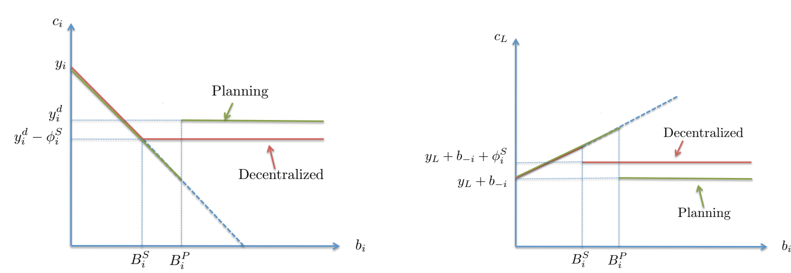
(a) Decentralized (b) Planning

Figure 1: Default and Repayment

The outcome of the planning problem is also similar to the decentralized problem in that default occurs in equilibrium. Even though default entails real resource costs, the planner prefers default when the debt of the country is sufficiently high. Defaults have redistribution benefits by reducing the transfers from the borrower to the lender. This result is related to that in D'Erasmo and Mendoza (2013), where default is precisely the government policy that allows redistribution from rich savers to poor borrowers. In our model, with stark linear payoffs, redistribution is valuable because the planner's weight on the borrower is larger than that of the lender. More generally, we think that the redistribution benefits of default can naturally arise due to risk-sharing reasons in richer models.

An important question from this analysis is whether default happens less in the planning problem than in the decentralized problem. The answer to this question depends on parameters. If default in the planning problem happens because the planner is forced to default, which implies that the cutoff $B_i^P = y_i - y_i^{nr}$, then default sets in the planning problem are smaller than default sets in the decentralized problem as $y_i - y_i^d + \lambda_L/(\lambda_B + \lambda_L)(y_i^d - y_i^{nr}) < y_i - y_i^{nr}$. The intuition is that default is not optimal for the planner for levels just above the threshold, and hence for levels of debt just below the threshold B_i^P , it is setting the recovery to the maximum. In the decentralized problem, recoveries are lower than the maximum, and hence the levels of debt above which the country defaults are lower.

If in the planning problem, however, default is interior and $B_i^P = \lambda_B(y_i - y_i^d)/(\lambda_B - \lambda_L)$ then the planning problem features smaller default sets than the decentralized problem only if $\left(y_i^d - y_i^{nr}\right)/(\lambda_B + \lambda_L) < (y_i - y_i^d)/(\lambda_B - \lambda_L)$. This condition says that the larger the deadweight cost from default or the smaller the weight of the borrower relative to the lender, the more likely the planners' default set is smaller than the decentralized default set. Such conditions are intuitive given that the benefits of default for the planner are the redistribution of resources from the lender to the borrower.



(a) Borrower (b) Lender

Figure 2: Consumption in Decentralized and Planning Problems

We now compare the consumption allocations across the planning problem and the decentralized problem. Figure 2 illustrates the consumption of borrower i and the lender across the two problems when default set is larger in the decentralized problem and and $B_i^P = \lambda_B(y_i - y_i^d)/(\lambda_B - \lambda_L)$. We consider the case in which the other country is repaying. For the region of debt $b_i < B_i^S$, the consumption allocations in both problems are equal because the borrower is repaying. For the region of $B_i^S < b_i < B_i^P$, the planner is inducing repayment, whereas in the decentralized model the country defaults. Here, in the planning problem consumption for the borrower is lower and that of the lender is higher than in the decentralized problem. In the region where $b_i > B_i^P$, default happens in both problems, but in the planning problem the consumption of the borrower is higher and that of the lender is lower. When default is optimal in the planning problem, it is optimal to set recovery to zero.

II. Conclusion

This paper has studied the role of policy in renegotiation protocols when multiple countries borrow from the same lenders. The paper compares a decentralized Nash bargaining protocol with one designed by a benevolent planner. In the decentralized model, a default in one country increases the likelihood of default for the second country because recoveries are lower when both countries renegotiate together with the lender. In the planning solution, in contrast, the defaults of each country are independent of the other country. The planner simply decides on recoveries that induce default or repayment of each country, trading off the deadweight costs and the redistribution benefits of default.

The paper has identified an important role for policy in preventing contagion of sovereign debt crises. These policies should be aimed at designing renegotiation processes that treat each country in isolation. Such policies contrast the common discussions in Europe that bundle potential defaults in one country to defaults in other countries. Our analysis suggests that these types of discussions might not be useful and that they precisely may exacerbate the coordination problems across countries. Our paper has also shown that avoiding default is not always optimal because default allows redistribution from lenders to borrowers even if default carries deadweight costs.

The design and implementation of bargaining protocols have precedence in other relations such as those between unions and firms. In the United States. the National Labor Relations Board is in charge of the rules and regulations governing how workers and firms interact. Our work suggests that it would be useful for the European Union to design and enforce renegotiation protocols between borrowing countries and lenders that treat each pair in isolation.

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Appendix

Proposition 1. *Country i defaults in the planning problem iff $b_i \geq B_i^P$ where*

$$B_i^P \;=\; \min\left\{\frac{\lambda_B}{\lambda_B-\lambda_L}(y_i-y_i^d), \quad y_i-y_i^{nr}\right\} \text{ for } i=1,2$$

Proof. We prove this proposition by contradiction. Suppose that $b_1 < B_1^P$ but country 1 defaults in the planning problem. This can only be a solution if it is feasible for the planner to induce country 1 to default and such strategy increases the value for the planner.

To induce default, the planner needs to set the recovery low enough, $\tilde{\phi}_1 < \phi_1^*$ where $y_1^d - \phi_1^* = y_1 - b_1$. If $\phi_1^* \geq 0$, it is feasible for the planner to induce default independent of the default outcome for country 2 because $\tilde{\phi}_1 = 0$ is always feasible. However, inducing country 1 to default is not optimal because $b_1 < \frac{\lambda_B}{\lambda_B-\lambda_L}(y_1-y_1^d)$ which implies that the additional value from country i defaulting is negative $\lambda_B(y_1^d - y_1) + (\lambda_B-\lambda_L)b_1 < 0$.

Now, consider the more interesting case of $\phi_1^* < 0$. The planner can only induce a default in country 1 by setting $\tilde{\phi}_1 = \phi_1^* < 0$ which requires that country 2 also defaults and pays a recovery $\tilde{\phi}_2 > 0$ that is high enough. By Lemma 3, it is optimal to set $\tilde{\phi}_2 = -\tilde{\phi}_1 = -\phi_1^* > 0$.

Suppose that $\tilde{\phi}_2 = -\phi_1^*$ does not induce country 2 to default $y_2^d - \tilde{\phi}_2 < y_2 - b_2$. For such $\tilde{\phi}_2$, the planner cannot induce country 1 to default because the lower bound on the recovery for country 1 $\underline{\phi}_1 = 0$ is too high.

Suppose now that we make $\tilde{\phi}_2 = y_2^d - y_2 + b_2$ to induce country 2 to default and $\tilde{\phi}_1 = -\tilde{\phi}_2$. Here we have two cases. If $b_2 < B_2^P$ then the gains for the planner for additional two defaults are: $[\lambda_B(y_1^d - y_1) + (\lambda_B-\lambda_L)(b_1+\tilde{\phi}_2) + \lambda_B(y_2^d - y_2) + (\lambda_B-\lambda_L)(b_2-\tilde{\phi}_2)] = [\lambda_B(y_1^d - y_1) + (\lambda_B-\lambda_L)b_1 + \lambda_B(y_2^d - y_2) + (\lambda_B-\lambda_L)b_2] < 0$ because $b_1 < B_1^P$ and $b_2 < B_2^P$.

If $b_2 > B_2^P$, then $\tilde{\phi}_2 > 0$ could continue to induce 2 default. The gains for the planner from the additional country 1 default are: $[\lambda_B(y_1^d - y_1) + (\lambda_B-\lambda_L)(b_1+\tilde{\phi}_2) + \lambda_B(y_2^d - y_2) + (\lambda_B-\lambda_L)(b_2-\tilde{\phi}_2)] - [\lambda_B(y_2^d - y_2) + (\lambda_B-\lambda_L)b_2] = \lambda_B(y_1^d - y_1) + (\lambda_B-\lambda_L)b_1 < 0$ for $b_1 < B_1^P$ where we have used Lemma 3 that when only one country defaults it is optimal to have

$\phi_2^P = 0$. Hence having a default for country 1 when $b_1 < B_1^P$ does not increase the value for the planner. $\square$