

# Partial Default

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# Motivation

Understanding debt crises important for emerging markets

In data sovereign default episodes have rich dynamics – [Default leads to more default](#)

- ▶ Defaults are partial, countries spend much time in default
- ▶ Debt increases during default episodes
- ▶ Repayment, borrowing, more default during episodes

Existing sovereign default theory – [Default leads to new start](#)

- ▶ Default episodes leads to reduction in debt (zero in many models)
- ▶ During default episodes no borrowing or repayment
- ▶ Cannot accommodate these properties

New theory of sovereign default with rich dynamics as in data

# Sovereign Defaults in Data

Defaults are partial, frequent, and of varying duration (much heterogeneity)

- ▶ Partial default on average 35%, 1/3 the time positive, large variance
- ▶ Larger defaults associated with higher spreads, higher debt, lower output

Default episodes

- ▶ On average 9 years long, but with many short default episodes
- ▶ Debt and partial default dynamics hump-shaped
- ▶ Debt not reduced but sizable haircuts

# Model Elements

- ▶ Sovereign chooses to which extent to pay debt due
- ▶ Partially defaulted debt not automatically written off, nor new borrowing ruled out
- ▶ Partial default is alternative to borrowing to inter-temporally transfer resources

## Partial default as portfolio choice

- ▶ Partial default amplifies debt crises
  - ▶ Defaulted payments accumulate and increase future indebtedness
  - ▶ Associated with resource costs and higher spreads on new borrowing

## Default leads to more default

- ▶ Default episodes end after sufficient output recovery and deleveraging
  - ▶ Tight bond prices gives incentive for deleveraging

# Main Findings

This theory is capable of rationalizing patterns in data

- ▶ Properties of partial default: frequency, mean, and co-movements
  - ▶ Large partial defaults → high spreads, high debt, deep recessions, and longer
- ▶ Default episodes that resemble data
  - ▶ Long episodes on average, with many short
  - ▶ Deliver hump-shaped patterns for debt and partial default
  - ▶ Not resulting in a net reduction of debt but with sizable haircuts

Debt resolutions mechanisms

- ▶ Debt relief and bond covenants less useful than in standard theory

# Existing Literature

- ▶ Sovereign default theory with repaying or defaulting in full – extreme default (Eaton-Gersovitz 1983, Aguiar-Gopinath 2006, Arellano 2008, Chatterjee-Eyigungor 2012)  
[Here partial default, endogenous length of episode with rich dynamics](#)
- ▶ With countries restructuring all of its debt at the end of impasse (Bulow-Rogoff 1989, Yue 2010, Benjamin-Wright 2009, Asonuma-Joo 2020)  
[Here default episodes have dynamics for debt and partial default](#)
- ▶ In household default models, bankruptcy with fresh start (Chatterjee et al 2007, Livshits et al 2008)  
Recent interest in delinquent debt – partial default (Mateos-Planas-Seccia 2015, Herkenkoff 2013)
- ▶ Recent work with selective sovereign default but without amplification (Gordon-Gerron-Quintana 2019, Erce-Malluci 2018)  
[Here default leads to amplification of crisis](#)
- ▶ Empirical work documenting properties of default episodes (Cruces-Trebesch 2013, Asonuma-Trebesch 2016, Benjamin-Wright 2010, Tomz-Wright 2013)  
[Here macro approach with accounting framework and public datasets](#)

# Plan

- ▶ Empirical properties of sovereign defaults: accounting + data
- ▶ Model and mechanisms
- ▶ Quantitative analysis: fitting the model to some facts; accounting for other facts
- ▶ Policy counterfactuals: *pari passu*, debt relief, no dilution covenants

# Empirical Properties of Sovereign Defaults

- ▶ Develop accounting framework to analyze data
- ▶ Use panel data for 37 emerging countries 50 years
  - ▶ World Bank data (WDI, International Debt Statistics, Debtor Reporting System):  
public debt in arrears, debt service, debt levels, GDP
  - ▶ Global Financial Indicators on EMBI+ yield spreads
- ▶ Document properties of partial default and default episodes



# Accounting: Flows and Levels

- ▶ Each period sovereign owes lenders  $a_t$  - sum of coupons from past issuances
- ▶ Flexible partial default policy: sovereign pays  $(1 - d_t)a_t$  and does not pay  $d_t a_t$

$$\text{Debt service}_t = (1 - d_t)a_t$$

$$\text{Defaulted coupons}_t = d_t a_t$$

$$\text{Debt due}_t = \text{Debt service}_t + \text{Defaulted coupons}_t = a_t$$

$$\text{Partial default}_t = \text{Defaulted coupons}/\text{Debt due} = d_t$$

- ▶ Debt level is present value of future coupon promises

$$\text{Debt}_t = \sum_{j=0}^{\infty} \frac{a_t^{t+j}}{R^j} = \frac{a_t}{1 + r - \delta}$$

- ▶ Long-term perpetuity bonds that decay at rate  $\delta$  (Hatchondo-Martinez 09)

Sovereign receives  $q_t b_t$ , promises to pay, conditional on not defaulting,  $\delta^{n-1} b_t$  in  $t + n$

## Accounting: Default episodes and haircuts

- ▶ Default episode: Sequence of periods with consecutive positive partial default  
Episode of **length**  $N + 1$  has  $d_{t+j} > 0$  for  $j = \{0, 1, \dots, N\}$
- ▶ Defaulted coupons  $d_t a_t$  result in future obligations with PV  $\kappa d_t a_t$  (annuitize into perpetuity)  
Factor  $\kappa$  captures that sovereign accumulates debt in arrears and restructures

- ▶ Restructured new coupons in the episode  $n_t$  from accumulation of arrears

$$n_{t+j+1} = \begin{cases} (R - \delta)\kappa d_{t+j} a_{t+j} + \delta n_{t+j} & \text{for } j = \{0, 1, \dots, N\} \\ \delta^{j-N-1} n_{t+j} & \text{for } j = \{N + 1, \dots, \infty\} \end{cases}$$

- ▶ Result in new long-term restructured debt, present value of restructured coupons

$$\text{Restructured debt}_t = \sum_{j=1}^N \frac{(1 - d_{t+j}) n_{t+j}}{R^j} + n_{t+N+1} \sum_{j=0}^{\infty} \frac{\delta^j}{R^{N+j+1}}$$

- ▶ Defaulted debt is the present value of the defaulted coupons

$$\text{Defaulted debt}_t = \sum_{j=0}^N \frac{d_{t+j} a_{t+j}}{R^j}$$

# Accounting: Default episodes and haircuts

- ▶ Haircuts depend on value of the defaulted debt and the restructured debt (Cruces-Trebesch 13)

$$\text{Haircut}_t = 1 - \frac{\text{Restructured debt}_t}{\text{Defaulted debt}_t}$$

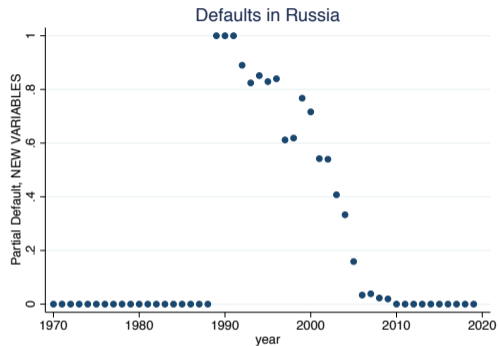
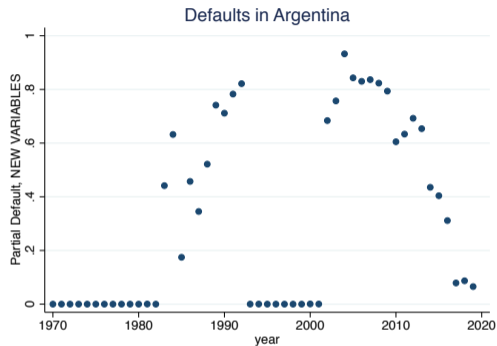
- ▶ In default episode sovereign carries its legacy debt due, restructured coupons, and borrowing
- ▶ A sovereign with debt due  $a_t$ , that borrows  $b_t$ , and partially defaults on  $d_t a_t$  has

$$a_{t+1} = \delta a_t + (R - \delta) \kappa d_t a_t + b_t.$$

- ▶ Accumulation of debt due incorporates all these
- ▶ Partial default and debt during the episode: sequences  $\{d_{t+j}, \text{Debt}_{t+j}\}$  for  $1 \leq j \leq N$

Use accounting framework to map data into our variables of interest

# Partial Default and Episodes Examples



- ▶ Partial default frequent, varies in intensity
- ▶ Argentina experienced two episodes with lengths equal to 10 and 18 years
- ▶ Russia experienced one 20 year episode

# Russia & Argentina: Some Details

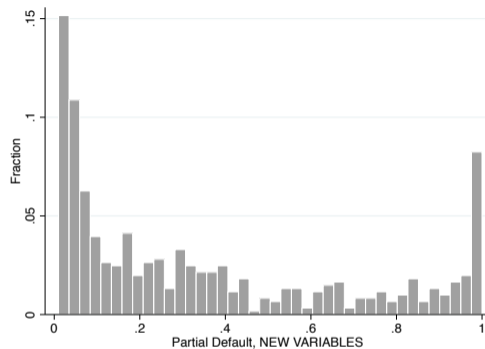
## Russian episode 1992-2012 (Santos 2003)

- ▶ Partial defaults and restructurings: multiple restructurings with Paris Club (93, 94, 95, 96, 99); London Club (97, 2000), COMECON 94, bond IAN, PRIN, MinFin II 1999, and trade creditors
- ▶ Borrowings: 1996 sizable Eurobond while in arrears with London Club; largest amount borrowings in 1998, new loans from each restructurings, loans from multilateral financial institutions (IMF)

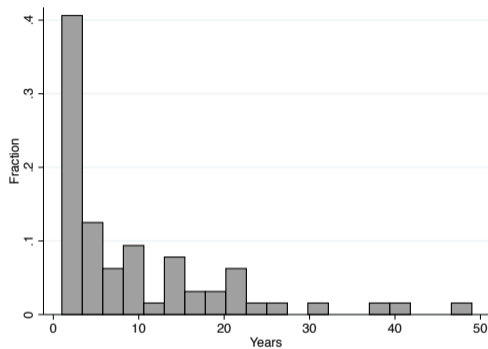
## Argentinian episode 2001-2019 (Horneck 2004, 18-K SEC Report 2020)

- ▶ Partial defaults and restructurings: Default of 2001 on most of private bond and loans and official bilateral loans
  - ▶ Prior to 2005 restructuring 44% debt was performing and not all the non-performing debt was included. 75% of creditors accepted 2005 exchange
  - ▶ 2010 exchange offer to holdout creditors, accepted by about 12%. Rest finally resolved in 2016 with litigation in court after pari pasu clause applied
  - ▶ Paris club debt in arrears throughout the period
- ▶ Borrowing during episode: Domestic debt in 2002 (Bonar) with estimate 70% foreign investors, loans from Inter-American Bank, 14 loans from Chinese government from 2006-2017, bonds as part of exchanges, large new international bond issuances 2016-2018

# Distribution of Partial Default and Episode Length



(a) Partial Default



(b) Default Episode Length

- ▶ Wide dispersion of partial default across countries and time
- ▶ Many small defaults: about 30% with partial default < 10%
- ▶ Many short default episodes and a few countries always have positive default

# Partial Default and Episodes across Countries

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<i>Partial Default Frequency   &gt; 0</i>			
Frequency			33
Mean			35
Standard deviation			16

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<i>Default Episodes</i>			
Episode length (years)			9
Fraction of short episodes ( $\leq 2$ years)			35
Haircut (%)			36
Maturity extension (years)			6

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<i>Default Episodes Dynamics</i>	Partial Default	Debt	Output
Before	0	32	0
Beginning	17	34	-1
Middle	28	40	-5
After	0	33	-3

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- ▶ Frequent partial default 1/3, with mean 35%
- ▶ Long default episodes on average, but lots of short
- ▶ Hump-shaped partial default and debt during default episodes; U shaped for output

# Partial Default Bins and Co-movements

Spreads, debt, and output by partial default quantiles

<i>Means (%)</i>	Partial default > 0			
	No default	Small (0 – 25%)	Medium (25 – 75%)	Large (25 – 75%)
Partial default	0	3	26	87
Spreads	3	6	8	19
Debt to output	25	33	43	56
Output	1	-1	-2	-5

- ▶ Larger partial defaults with: high spreads, & debt, lower output
- ▶ Longer default episodes also tend to have higher partial default



# Data Summary

1. Partial default: one third of time positive, on average 35%, large variance
2. With higher partial default: higher spreads and debt, more depressed output
3. Default episodes:
  - ▶ Hump-shape dynamics in partial default and debt, U shape for output
  - ▶ Do not lead to a net reduction in debt

# Model: Environment

- ▶ Small open economy with stochastic endowment  $z_t$
- ▶ Borrows internationally long-term perpetuity bonds with price  $q_t$
- ▶ Lacks commitment and can partially default on its coupons
  - ▶ Default reduces income: cost depend on intensity  $d_t$
  - ▶ Defaulted coupons accumulate with factor  $\kappa$
  - ▶ With partial default, sovereign can continue to borrow at market higher rates
- ▶ International lenders risk neutral, bond prices compensate to default risk

# Sovereign Borrower

- ▶ Preferences over consumption  $E \sum_{t=0}^{\infty} \beta^t u(c_t)$

- ▶ Consumption is income  $y_t$  net of repayment of debt service and borrowing

$$c_t = y_t - a_t(1 - d_t) + q(a_{t+1}, d_t, z_t)b_t$$

- ▶ Partial default  $d_t$  expands  $c_t$  but depresses income  $y_{t+1} = z_{t+1}\psi(d_t, z_{t+1}) \leq z_{t+1}$

Partial default and new borrowing is a portfolio problem

- ▶ Debt due = legacy debt  $\delta a_t$  + borrowing  $b_t$  +  $\kappa$  of new restructured  $(1 + r - \delta)\kappa d_t a_t$

$$a_{t+1} = \delta a_t + (R - \delta)\kappa d_t a_t + b_t$$

- ▶ Sovereign can always borrow, even with default but prices  $q(a_{t+1}, d_t, z_t)$  respond

$d_t$  lowers prices because it increases  $a_{t+1}$  and lowers  $y_{t+1}$

# Recursive Problem and Bond Price

- ▶ State is  $(a, y, z)$ :  $a$  debt due;  $y$  income (due to default history);  $z$  persistent productivity

$$V(a, y, z) = \max_{b, d \in [0, 1]} \{u(c) + \beta E_z V(a', y', z')\}$$

subject to budget constraint, accumulation debt due, and income transition

- ▶ No separate problem for “default” states
- ▶ Bond price compensates for partial default loss to competitive lenders that discount at  $r$

$$q(a', d, z) = \frac{1}{R} E \left( \underbrace{(1 - d(a', y', z'))}_{\text{partial default next}} + \left[ \delta + \underbrace{(R - \delta)\kappa d(a', y', z')}_{\text{new restructured}} \right] \underbrace{q(a'', d', z')}_{\text{partial default future}} \right)$$

# Trade-offs for Borrowing and Partial Default

A portfolio-choice

$$\begin{aligned} b : \quad & u_c[q + q_{a'}] && = \beta E u'_c \Lambda' \\ d : \quad & u_c[a + (q_{a'}(R - \delta)\kappa a + q_d)b] && = \beta E u'_c [(1 + r - \delta)\kappa a \Lambda' - z' \psi_d] \end{aligned}$$

where debt burden  $\Lambda' \equiv (1 - d') + (\delta + (R - \delta)\kappa d')q'$

- ▶ Equate expected returns  $R^b = R^d = u_c(c)/[\beta E u_c(c')]$

$$R^b \equiv \frac{R}{1 + q_{a'} b/q} + \text{cov}_1 = \frac{E z'(-\Psi_d)}{a(1 - q(R - \delta)\kappa) + q_d b} + \text{cov}_2 \equiv R^d$$

- ▶ Borrowing more attractive when  $q$  high and not too steep ( $R^b$  low)
- ▶ Partial default more attractive when  $a$  high,  $q$  low, default costs not too steep ( $R^d$  low)
- ▶ Steep bond price incentivizes exit from default episodes “induces deleveraging”

# Quantitative Analysis

- ▶ Parameterize model to time series data of partial default and debt
- ▶ Evaluate model performance for partial default co-movements and default episodes
- ▶ Perform counterfactuals: debt relief, bond covenants
- ▶ Compare with reference model

# Numerical Settings

- ▶ Functional forms:

$$\text{Utility: } u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

Default costs: output decreasing concave in  $d$   $y' = z' \Psi(z', d)$

$$\Psi(z', d) = (1 - \phi_0 d^\gamma) \times \begin{cases} 1 - \phi_1(z' - z^*) & d > 0, z' > z^* \\ 1 & \text{otherwise} \end{cases}$$

- ▶ Parameters: set  $\sigma = 2$ ,  $r = 4$ , estimate 9 parameters (default cost parameters, discount factor, recovery factor, shock process, bond decay) to target 11 moments:

properties of partial default, debt to output, debt service, and spreads

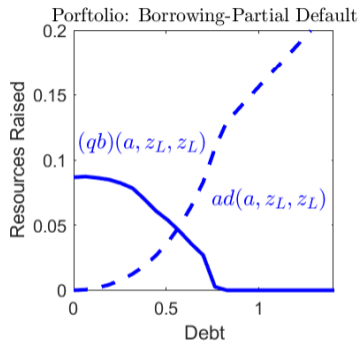
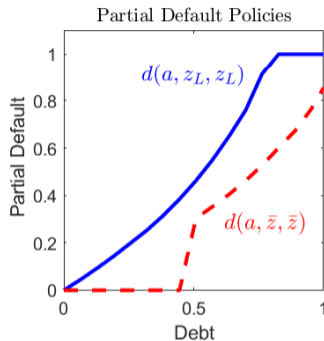
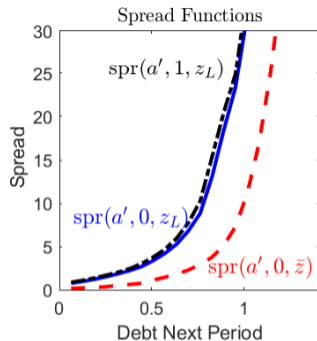
- ▶ Apply accounting framework to model for time series statistics, default episodes, haircuts

# Moment Matching

	Data	Model
<i>Target Moments</i>		
Partial default (in %)		
frequency	33	37
mean	35	39
st. dev.	16	19
Debt to output (in %)		
mean	32	32
st. dev.	18	25
Debt service to output (in %)		
mean	3.6	3.5
st. dev.	2.1	2.2
Debt due to output mean	4.8	5
Spread st. dev.	4.1	3.7
Output		
persistence	0.89	0.88
st. dev. (in %)	10	12
<i>Other Moments in Panel</i>		
Spreads		
mean	5.3	1.6
correlation with output	-17	-38
correlation with debt	24	56
Consumption st. dev. (relative to output)	1.0	0.91



# Mechanisms: Spreads, Partial Default, and Portfolio



- ▶ Spreads smooth, mostly depend on  $a'$  not on  $d$  directly but  $a' = \delta a + da\kappa(R - \delta) + b$
- ▶ Higher debt + high spread  $\rightarrow$  larger partial default
- ▶ With low debt + low spread  $\rightarrow$  borrow
- ▶ Partial default endogenously restrict new borrowing (default piles up debt due)

# Partial Default Co-movements: Model and Data

<i>Means (%)</i>	Partial default bins			
	No default	Small	Medium	Large
	<i>Data</i>			
Partial default	0	3	26	87
Debt to output	25	33	43	56
Spreads	3	6	8	19
Output	1	-1	-2	-5
	<i>Model</i>			
Partial default	0	20	35	66
Debt to output	18	33	55	82
Spreads	1	1	2	8
Output	6	-11	-10	-18

- ▶ Model matches untargeted co-movements
- ▶ Large partial defaults associated with: high spreads, high debt, low output

# Default Episodes in Model and Data

	Data	Model
Mean episode length (years)	9	8
Percentage of short episodes ( $\leq 2$ )	35	42
Coefficient of variation for episode length	1.2	1.5
Haircut (%)	36	37
Maturity extension	6	7

Good fit for default episodes

- ▶ Model delivers long default episodes on average, with many short too
- ▶ Delivers haircuts and maturity extensions that resemble data

# Default Episodes Dynamics in Model and Data

	Data	Model
Partial Default		
Before	0	0
Beginning	17	21
Middle	28	28
End	0	0
Output		
Before	0	0
Beginning	-1	-7
Middle	-5	-9
End	-3	3
Debt		
Before	32	32
Beginning	34	35
Middle	40	44
End	33	42

- ▶ Default leads to more default, no reduction in debt
- ▶ Hump shape dynamics for partial default and debt; U shape for output

# Resolution Mechanisms Counterfactuals

## PARI PASSU: NO MARKET ACCESS DURING DEFAULT

- ▶ Smaller haircuts on more recent issuances breaches pari passu (or equal treatment) clauses
- ▶ Pari passu might impede borrowing in default
- ▶ Defaulting becomes more costly, but credit is easier

## DEBT RELIEF

- ▶ A permanent reduction in recovery factor  $\kappa$
- ▶ Defaulting becomes less costly, but credit is harder

## NO DILUTION COVENANTS

- ▶ Add no dilution covenants of Hatchondo-Martinez-SosaPadilla 2016
- ▶ Defaulting more costly, but credit easier

	Baseline	Pari Passu	Debt Relief	No-Dilution
<i>Default Episodes</i>				
Mean episode length (years)	8	2	8	6
Percentage of short episodes ( $\leq 2$ )	42	89	42	41
Coefficient of variation for episode length	1.5	0.8	1.4	1.2
Haircut (%)	37	32	46	36
<i>Time series in (%)</i>				
Partial default				
frequency	37	11	35	31
mean	39	44	36	31
st. dev.	19	15	16	10
Debt to output mean	32	27	23	26
Spread st. dev.	3.7	1.1	2.3	0.9
<i>Welfare rel. baseline (% CE)</i>				
No debt, $z_L$	–	0.12	-0.11	0.07
Debt 64%, $z_L$	–	-0.07	0.05	-0.20
Partial default= 0, average	–	0.05	-0.05	0.02
Partial default> 0, average	–	-0.01	-0.02	-0.05
Overall Average	–	0.03	-0.04	-0.01

- ▶ Pari Passu: Less frequent partial default, shorter episodes, improves welfare when debt low
- ▶ Debt Relief: Larger haircuts and lower debt, improves welfare when debt high
- ▶ No Dilution: Lower debt, reduced partial default, improves welfare when debt low

# Resolution Mechanisms Counterfactuals

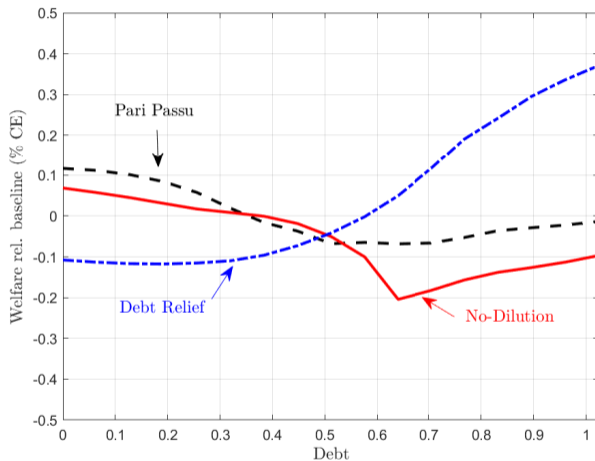


Figure: Relative Welfare from Counterfactuals for  $z_L$

# Reference Model: Renegotiation with Bargaining

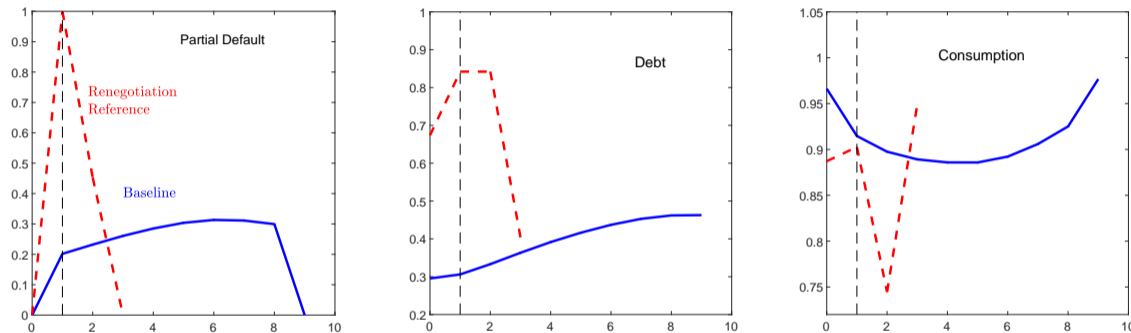


Figure: Default Episodes in Baseline and Reference

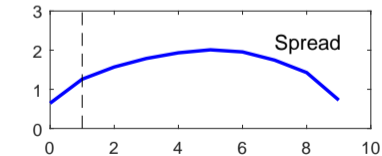
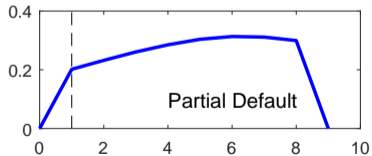
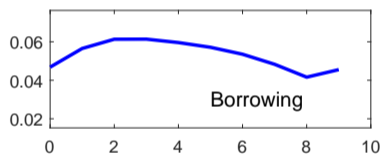
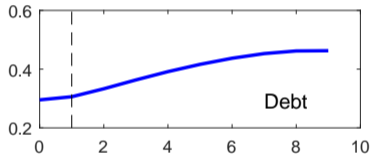
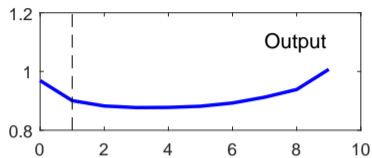
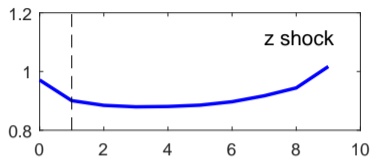
- ▶ Default episodes short in reference model
- ▶ Debt collapses upon default + burst borrowing and consumption upon re-entry: default leads to fresh start
- ▶ Misses debt dynamics and amplification of default episodes



# Conclusion

- ▶ Prior work focuses on dynamics prior to default, with fresh start after default
- ▶ Document rich dynamics during default and amplification of default
- ▶ Propose a theory with partial default: default leads to more default
  - ▶ Useful for rationalizing dynamics during default episodes, properties of partial default
- ▶ Theory potentially useful to analyze restructuring mechanisms
  - ▶ That can actually lead to a reduction in debt burden

# Default Episodes in Model



# Reference Model Comparison

	Data	Baseline	Renegotiation
<i>Target Moments (in %)</i>			
Partial default			
frequency	33	37	3
mean	35	39	76
st. dev.	16	19	26
Debt to output			
mean	32	32	30
st. dev.	18	25	14
Debt service to output			
mean	3.6	3.5	4.4
st. dev.	2.1	2.2	1.6
Debt due to output mean	4.8	5.0	4.7
Spread st. dev.	4.1	3.7	6.0
<i>Properties of Episodes</i>			
Mean episode length (years)	9	8	2.01
Percentage of short episodes ( $\leq 2$ )	35	42	98.9
Coefficient of variation for episode length	1.2	1.5	0.07
Haircut (%)	36	37	54