Partial Default
Online Appendix*

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A Data Sources and Countries

Our sample of countries consists of all 37 emerging countries from the J.P. Morgan Emerging Market Bond Index (EMBI+). The countries in the sample are Argentina, Bulgaria, Belize, Brazil, Chile, the Dominican Republic, Ecuador, Gabon, Ghana, Indonesia, Jamaica, Morocco, Mexico, Nigeria, Pakistan, Panama, Peru, the Philippines, Poland, the Russian Federation, El Salvador, Serbia, Trinidad and Tobago, Turkey, Ukraine, Uruguay, Venezuela, Vietnam, South Africa, China, Colombia, Egypt, Hungary, Korea, Sri Lanka, Malaysia, and Tunisia. The data are annual and come from the World Development Indicators (WDI), the International Debt Statistics, the Debtor Reporting System, and the Global Financial Database (GFD). The data span 1970 to 2019, and the time series coverage for each variable is based on availability. As explained in Section 2.2, we use the following debt variables for public and publicly guaranteed (PPG): debt service on external debt, external debt stocks, interest arrears, and principal arrears.[1] We also use EMBI+ spreads, real and nominal (in dollars) gross domestic product, and real private consumption.

B Alternative Measure of Haircuts

In our baseline results, we rely on the empirical measures for haircuts from default episodes from Cruces and Trebesch (2013). In this appendix, we use our dataset on defaulted coupons and default episode timing to compute haircuts using our accounting framework at the estimated parameters. We apply the formula for haircuts (9) using the time series for defaulted coupons (2) by constructing the values for defaulted debt and restructured debt, as in our accounting framework, using the estimated values for the parameters that enter into the calculation, namely $\{R, \kappa, \delta\}$. We find that across the 63 default episodes in our data, the median haircut is 33%, and the mean haircut is 40%. These values are similar to the average haircut of 36% reported in Cruces and Trebesch (2013).

C Calibration Model with Renegotiation

The penalty for default $\Psi(z)$ takes on the same functional form as for the baseline partial-default model $\Psi(d, z)$ in (23) when evaluated only at $d = 1$, since default is full here. The parameter γ therefore plays no role. The utility function $u(.)$ is also unchanged.

[1] Owing to a lack of data availability for PPG, for Venezuela, we use interest and principal arrears long-term (DOD) to estimate the PPG series because these two series feature very high correlation in other countries.

The process for the identity of the proposer is fully characterized by the probability of the sovereign proposing π_s.

We again set directly the values of risk aversion σ and the risk-free interest rate R. The size of the extreme-value shocks affecting the binary choices of the sovereign, σ_{EV}, and the lenders, σ_{EV}^{ℓ}, are set to facilitate convergence. At values $\sigma_{EV}^{s} = 0.01$ and $\sigma_{EV}^{\ell} = 0.001$, for our purpose, they do not appear to play a role in this model.[2]

The remaining eight parameters, $\Theta^R = \{\phi_0, \phi_1, z^*, \beta, \pi_s, \delta, \rho, \sigma_\eta\}$, are estimated by minimizing the sum of the proportional square residuals of the same 11 moments from data as in the estimation of the partial-default model. Tab. 1 displays the parameters obtained. The outcomes for the

Table 1: Renegotiation Model - Estimated Parameters

Default costs	$\phi_0 = 0.0995$
	$\phi_1 = 0.0979$
	$z^* = 0.970 \times \bar{z}$
Discount factor	$\beta = 0.937$
Decay parameter annual	$\delta = 0.877$
Sovereign's offer making probability	$\pi_s = 0.479$
Shock process	$\rho = 0.865$
	$\sigma_\eta = 0.051$

bargaining model reported in Section 6 are based on these values. As shown there, this setting implies default episodes with an average length of about two years.

The literature, like the contributions of Asonuma and Joo (2020) and Dvorkin et al. (2021), assumes that there is no output cost if the realization of the endowment is sufficiently small. This amounts to imposing $\phi_0 = 0$ in our specification of Ψ. We have also estimated the renegotiation model under this restriction, with the same targets as above. The resulting parameter values are displayed in Table 2.

Table 3 displays some of the targeted moments and episode length under these two calibrations. Restricting $\phi_0 = 0$, predictably, implies a poorer fit to moments overall and a β that is implausibly low. It raises the length of episodes, although only to a small extent.

[2] We have calculated false default and false no-default binary decisions. False defaults are defaults that occur when their probability is less than 0.5; false no-defaults are situations in which default does not happen, although its probability is greater than 0.5. We obtain that 2.7% of all defaults are false and that 0.023% of all no-defaults are false. In any case, extreme-value shocks would if anything contribute to making episodes longer and more meaningful in this model.

Table 2: Renegotiation Model - Est. Parameters when $\phi_0 = 0$

Default costs	$\phi_0 = 0.0000$
	$\phi_1 = 0.3759$
	$z^* = 0.899 \times \bar{z}$
Discount factor	$\beta = 0.915$
Decay parameter annual	$\delta = 0.877$
Sovereign's offer making probability	$\pi_s = 0.471$
Shock process	$\rho = 0.831$
	$\sigma_\eta = 0.049$

Table 3: Renegotiation Model - Implications

	ϕ_0 estimated	$\phi_0 = 0$ set
Partial default	0.76	0.76
Frequency	0.03	0.67
Stand Dev partial default	0.26	0.29
Debt due/output	0.047	0.069
Stand Dev spread	0.060	0.085
Haircut	0.54	0.46
Episode length	2.01	2.50

D Appendix: Impulse Responses

We analyze here the equilibrium time series of partial default and spreads, as well as borrowing, total debt due, and output, in response to shocks with impulse response functions. We construct impulse response functions in our nonlinear model as follows. We simulate $1,000,000$ paths for 600 periods. From periods 1 to 500, the aggregate shocks follow their underlying Markov chains. In period 501, we reduce the value of productivity z in each simulation, and from then on they follow the conditional Markov chains. The impulse responses plot the average, across the $1,000,000$ paths, of the variables from period 500 to 600. Figure 1 contains these responses for a small shock of half a standard deviation (solid lines) and a large shock of one standard deviation (dashed lines) shock in period 0. The top left panel of the figure plots the paths for the shocks z in percentage deviations. The paths for output essentially mirror the paths for productivity as the impact of default costs is not large.[3] The top right panel shows the path for consumption, which on impact falls a bit less than output, but it stays depressed longer than output does. Default risk restricts the ability of the economy to smooth consumption, leading to a large decline on impact.

[3] Default costs in these responses are very minor, 0.1% at most.

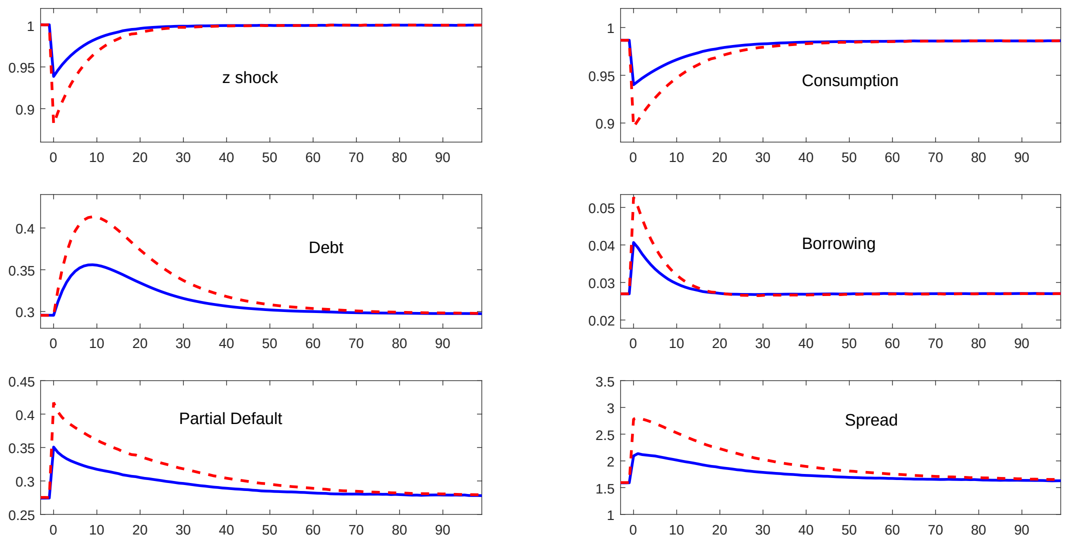

Figure 1: Impulse Response Functions

Consumption remains depressed because debt accumulates and remains persistently high.

The middle panels of Figure 1 plot the paths for the portfolio choices. Partial default increases on impact and features a response that is more persistent than the shock. By period 30, the shocks have largely recovered, while partial default remains above trend. Borrowing (reported relative to mean output) also increases on impact, but the effect is much less persistent than the effect on partial default.

The bottom left panel plots the response for debt to output that results from the choices of partial default and borrowing. The stock of debt features a hump shape, it increases for the first 10 periods after the shock and then slowly decreases. The rise in both partial default and borrowing increases the debt level. Partial default also delays the repayment of debt through the accumulation of defaulted coupons. The impulse response of total debt is slow and quite persistent. Debt remains elevated for much longer after the shock has returned to its mean. Finally, the bottom right panel in the figure plots the path for the interest rate spread. Spreads rise on impact about 60 basis points and decrease slowly. The effects on spreads are also very persistent.

The impulse response functions to a larger negative shock not only have larger amplitude but also lead to more persistent responses. The driver for more persistence is the more persistent debt dynamics.[4]

[4] The impulse response functions to positive shocks are almost exactly the mirror image of the impulse response functions to negative shocks. Only the dynamics of spreads are asymmetric. Booms feature smaller reductions in

E Computation

Baseline Model

In the model, the sovereign's decisions $b(a, y, z)$ and $d(a, y, z)$ and debt prices $q(a', d, z)$ are functions of the continuous endogenous state variables (a, y) and decision variables (a', d), respectively. To solve the model, we proceed by successive approximations of the value functions V for the sovereign and for the value of the debt H. We start by posing a grid in the state space. Then, given continuation values for the value function and for the value of debt, we solve for the optimal behavior conditional on defaulting and on not defaulting on each point of the grid using bilinear interpolation. Solving the problem of the sovereign requires the evaluation of price function q, which in turn involves bilinear interpolation of H. Next, the value of the choices conditional on defaulting and not defaulting are compared, and the highest value is chosen to update functions V and H on the grid.

Simulating the model involves trickier steps, as the sovereign choices are not in the grid. Realized states lie within rectangles that are surrounded by grid points. In some cases, in all the surrounding grid points, the decision is the same in terms of whether to default, in which case we simply interpolate the decision rules conditional on the same default binary choice at all the surrounding grid points. When this is not the case, we use for interpolation not the stored values of the value functions but the stored values of defaulting or not defaulting in all vertices of the rectangle. Then, choose the one that yields the highest value and interpolate the implied decision rules. This is particularly relevant, given that the nonconvex nature of the output costs when in default may yield the possibility that default starts at a non-zero threshold.

Robustness We tried other interpolation methods without any discernible difference. We increased the density of the grids until additional increases left the results practically unchanged.

Reference Model

We break down the equilibrium into two interdependent blocks. The first block determines outcomes conditional on the bad-standing policy rules for whether to reach agreement and the terms of the agreement. The second block studies the determination of these bad-standing policy rules given the outcomes from the first block. For the first block, we look for a solution to the recur-

spreads.

sive representation of the sovereign's problem. The decision on borrowing when not defaulting is represented as a continuous choice using splines to evaluate continuation values and debt prices outside of the grid. Continuation values for the lenders are evaluated using linear interpolation, since cubic splines may produce internodal oscillations where the slope becomes too steep. Given the binary default choice in the present case, we introduce small Type-I extreme-value shocks for this choice to help convergence.

Regarding the outcomes in the second block, when the sovereign proposes, we restrict new debt and repayment proposed to be non-negative, and when the lender proposes, we restrict the new coupon not to exceed the due repayment outstanding. As in Hatchondo, Martinez, and Sosa-Padilla (2016), we deal with explosive debt by preventing choices of levels of borrowing that imply an inordinately large spread. We solve the optimal proposal problems using optimality conditions rather than implementing direct constrained optimisation. The theory tells us that the proposer will in general, but not always, be offering terms on the other agent's participation constraint, and we use bisection to characterise the solution with appropriate bracketing to deal with cases in which there are more than two roots. When the solution does not correspond to a zero of the participation constraint, we determine the conditions for it to coincide with zero or with the maximum of the value to the lender (i.e., the top of the Laffer curve). For the binary outcome on whether there is agreement, we introduce extreme-value shocks under either proposer. We check ex-post that the frequency of actions that are not the optimal is small, as reported in Footnote 38 .

F Model with Binary Default Restriction

In our baseline model, the inclusion of default that is partial adds a finer instrument to adjust repayments with endowments realizations. In this section, we explore its significance by conducting a comparison of our model with a version of it that imposes a binary default decision $d = \{0, 1\}$, while keeping all other parameters as in the benchmark. The main takeaway is that partial default hurts ex-ante spreads but gives more state contingency. Tab. 4 below compares the main quantitative results of our model with binary default with those of the baseline partial default model.

With binary default, the frequency of default is lowered from 37% in the baseline to 4%, and the mean default is of course 100%. Default episodes are a bit shorter, and debt to output increases on average about 10%. With binary default, default costs are effectively higher in this economy, because the borrower cannot modulate them with a partial default decision. Higher default costs deter defaults and therefore make price schedules more favorable. In equilibrium this results in higher debt to output. The table also reports consumption equivalence welfare relative to that of the baseline model. The economy with binary default has higher average consumption equivalence welfare of 0.1%, where the average is taken across the debt distribution in the baseline economy. Welfare is on average higher with binary default because of the more favorable bond prices on average. Welfare can be lower with binary default in states where the benefits of state contingency of partial default are large enough. This situation tends to happen for levels of debt with positive but interior partial default.

Table 4: Binary Default Comparison

	Baseline	Binary Default
Default Episodes		
Mean episode length (years)	8	6
Coefficient of variation for episode length	1.5	2.0
Haircut (%)	37	44
Maturity extension	7	9
Time series in (%)		
Partial default		
frequency	37	4
mean	39	100
st. dev.	19	0
Debt to output mean	32	42
Spread st. dev.	3.7	3.2
Welfare rel. baseline (% CE)		
No debt, z_M	–	0.18
Debt 85%, z_M	–	-0.05
Overall Average	–	0.10

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