Monetary Policy and Sovereign Risk in Emerging Economies (NK-Default)*

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Abstract

This paper develops a New Keynesian model with sovereign debt and default. We focus on domestic interest rules governing monetary policy and external foreign currency government debt that is defaultable. Monetary policy and default risk interact as they both impact domestic consumption and production. We find that default risk generates monetary frictions, which amplify the monetary response to shocks. Large sovereign default risk depresses domestic consumption and production. These monetary frictions in turn discipline sovereign borrowing, resulting in slower debt accumulation and lower spreads. Our framework replicates the positive co-movements of sovereign spreads with domestic nominal rates and inflation, a salient feature of emerging markets data, and can rationalize the experience of Brazil during the 2015 downturn, with high inflation, nominal rates, and sovereign spreads. A counterfactual experiment shows that, by raising the domestic rate, the Brazilian central bank not only reduced inflation but also alleviated the debt crisis.

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1 Introduction

Since the early 2000s, following the steps of advanced economies, many central banks in emerging markets have achieved independence from the central government and have conquered their historical high inflation. Monetary policy has largely taken the form of interest rate rules that target inflation and has been guided by New Keynesian monetary models. Emerging markets also often struggle with government debt crises, which feature high interest rate spreads as well as outright defaults. Despite sovereign debt and monetary policy being important for consumption and output dynamics, New Keynesian theory has abstracted from default risk and most sovereign default theory has abstracted from nominal rigidities.[1] In this paper, we develop a New Keynesian model with sovereign default risk and identify important interactions. Default risk generates monetary frictions which amplify monetary responses to shocks, yet these frictions discipline the sovereign and slow down debt accumulation.

Our framework combines the workhorse New Keynesian monetary model of Gali and Monacelli (2005) with a standard sovereign default model. Default risk shapes monetary distortions and monetary policy. High default risk leads to low domestic consumption and production and is associated with large monetary distortions. These forces amplify the volatility of inflation and domestic nominal rates. Monetary frictions also impact government borrowing and default risk. Large monetary frictions curb government borrowing and lead to lower sovereign spreads. Our framework can deliver the positive correlations of sovereign spreads with inflation and domestic nominal rates present in emerging market data.

The small open economy model we consider consists of households, firms, a monetary authority, and a fiscal government that borrows internationally. Households value consumption of domestic goods and imported foreign goods. They work in intermediate goods firms that produce domestic varieties. The intermediate goods firms face frictions in setting their prices and are subject to productivity shocks. Firms have to pay an adjustment cost whenever they change their prices, in the tradition of Rotemberg (1982). Final goods firms are competitive and use intermediate goods varieties to produce domestic output, which is both consumed by domestic households and exported to the rest of the world.

The monetary authority sets policy by following a nominal interest rate rule, that depends on the gap between inflation and its target. Monetary policy is subject to shocks, so that the interest rate can deviate from the prescriptions of the rule. The fiscal government borrows from the rest of the world by issuing long-term bonds denominated in foreign currency and transfers the proceeds from these operations to households. The government lacks commitment over repaying the debt and can choose to default. Default is associated with a decline in productivity, which induces declines in consumption and production. The price of bonds reflects the risk of default.

[1]For example, the influential paper by Gali and Monacelli (2005) analyzes monetary policy in the context of perfect financial markets and the work of Aguiar and Gopinath (2006) and Arellano (2008) study sovereign default in real models.

As in standard New Keynesian models, price rigidities coupled with domestic interest rate rules generate *monetary frictions* that are reflected in firms' mark-ups or labor wedges. These monetary frictions are summarized by the New Keynesian Philips Curve where inflation decisions reflect that wedge between the marginal rate of substitution between consumption and leisure for workers and the marginal product of labor for firms. These frictions are state dependent. Contractionary monetary shocks, for example, trigger positive monetary frictions in our model; high nominal rates depress domestic consumption and production, leading to an increase in monetary frictions and a worsening in the labor wedge.

We consider a Markov problem for the fiscal government. The government internalizes that its borrowing and default decisions induce an allocation for the private economy, a monetary policy response, and also shape future default. Importantly, borrowing and default decisions affect the monetary friction, which induces state dependent deviations from efficient production. We show that default risk, arising from large borrowing, worsens monetary frictions and generates a decline in production. The reason is that an expectation of high default tomorrow lowers expected domestic consumption, which in turn lowers current domestic consumption through the domestic Euler equation because nominal rates do not change much due to price rigidities. A low consumption depresses production and increases monetary frictions.

Monetary frictions also affect sovereign borrowing incentives. We show that in our model, monetary frictions, associated with a low production, act as a tax on borrowing, and hence discourage borrowing. The larger monetary frictions that would arise if the government were to choose a higher borrowing are precisely the forces that discipline government's borrowing. Curbing borrowing is useful in our model because long-term defaultable bonds give the government incentives to overborrow and dilute existing bondholders.[2]

We derive an optimal borrowing condition that reflects the government's trade-offs. Borrowing in our model smooths fluctuations of consumption from imported goods, as in Gali and Monacelli (2005), but is modified to include *default* and *monetary* wedges. Default wedges arise because the price of bonds depends on borrowing due to default risk and also because such change in prices dilute the legacy debt. Monetary wedges reflect the interaction between government borrowing and default risk with monetary frictions. The choice of borrowing responds to all of these forces.

We parameterize our model to Brazilian data and perform a quantitative evaluation of our model. We set parameters controlling default costs and the interest rate rule as well as the volatility of shocks such that our model replicates the volatility of sovereign spreads, inflation, and output.

We show that our model can match the target moments well, and contains additional implications consistent with the data. Our model delivers strong positive comovement of spreads with nominal interest rates and inflation as in the data. We focus on these correlations

[2]Hatchondo et al. (2016) find that such dilution incentives are large and important for explaining the sizable spreads in emerging economies.

because such co-movement is a salient feature of emerging market data. We document that the positive correlations of spreads with inflation and nominal rates is a robust feature in 10 emerging markets with central banks that target inflation.

The model is also consistent with data in predicting additional co-movements with spreads: a negative correlation with output, and positive ones with the trade balance and the nominal exchange rate. The magnitudes of all of the correlations with spreads are similar to those in the data. Our model also generates co-movements with output consistent with data. Output is negatively correlated with inflation, nominal rates, the trade balance, and the nominal exchange rate in both model and data, although these correlations in the model are somewhat stronger.

To study the mechanisms generating these results and the interactions between monetary frictions and default, we compare our model to two reference models: a monetary model without sovereign default risk, similar the standard model of Gali and Monacelli (2005), and a real model with default in the tradition of the sovereign default literature, but with two goods and production.

We analyze the impact that government debt and default have on monetary frictions by comparing our benchmark model to the Gali and Monacelli (2005) model. We find that the labor wedge in our model depends on the level of government debt, but its relation depends on default risk. When default risk is low, the monetary frictions decreases with debt. Bond price schedules, that act as borrowing constraints, imply that capital inflows decrease with the debt. The economy works more to export and payoff the debt and the labor wedge falls. When debt is sufficiently high, however, and default risk is high and rising, the monetary frictions worsen rapidly with debt. Rising default risk depresses domestic consumption leading to depressed production. These forces translate into a response of inflation and nominal rates to debt. In contrast, in the reference monetary model, the labor wedge and nominal rates do not vary with foreign debt because borrowing is always ample. Such different dynamics lead to higher volatility of nominal rates in our benchmark model with default risk.

We then analyze the impact that monetary frictions have on government borrowing and spreads by comparing the accumulation of debt and the spread schedules in our model relative to the real reference model. We find that debt accumulation is slower in our model with monetary frictions, which lead to looser bond price schedules. These disciplining monetary forces lead to lower average spreads and default probabilities in our benchmark that in the real model.

We present impulse response functions to productivity and monetary shocks and compare them to the monetary reference model. In response to a contractionary monetary shock, inflation, output, domestic consumption, and borrowing fall on impact in both models because of the standard channels with price frictions. Nominal rates in our model rise by more than in the reference model because inflation falls by less. Government spreads fall in our model in response to contractionary shock because the monetary friction induce a decline in borrowing. The reference model exhibits no spreads and in contrast to our model presents an increase in borrowing.

These impulse responses illustrate the disciplining forces of monetary frictions. A high nominal rate increases monetary frictions and induces a reduction in borrowing and spreads. They also show that with default risk, contractionary monetary policy is less powerful in bringing down inflation as inflation falls less in the benchmark than in the reference despite nominal interest rates increasing by more in the benchmark.

In response to a low productivity shock, consumption and output fall, while spreads, inflation, and nominal interest rates rise. The increases in inflation and nominal rates are higher in the benchmark than in the reference model. This amplification arises in the benchmark because the bond price schedule is tighter with low productivity leading to an increase in spreads. The tight borrowing conditions reduce imports and stimulate exports, to pay off the debt. Inflation then increases by more because of a higher unit cost of production with the additional exports. In the reference model without default the economy can smooth the shock by taking on additional debt without reducing imports or increasing exports. The impulse responses to a productivity shock show that with default risk, nominal interest rates and inflation are more volatile. Pursuing an inflation target requires more aggressive movements in nominal rates in environments with default risk.

Finally, we perform an event analysis and evaluate a counterfactual monetary policy scenario. We focus on the period from 2014 to 2016. During the event, output fell in Brazil about 6% below trend, inflation and nominal interest rates increased about 4%, and spreads increased about 3%. We apply our model to this event by feeding the model a sequence of productivity shocks such that it reproduces the dynamics of output. We then compare the model implications for inflation, nominal rates, and spreads to the data. Our model reproduces sizable increases in inflation, nominal rates and spreads. Nominal interest rates increase as the inflation target rule calls for such tightening.

We then perform a counterfactual experiment that considers looser monetary policy during the event. We feed in the same sequence of productivity shocks but impose that the monetary authority does not increase nominal rates. We then compare the paths of inflation, spreads, and output in the counterfactual scenario to the benchmark. In the counterfactual, inflation increases more, output decreases less, and spread increase substantially more. We conclude that the increase in nominal rates in Brazil, during the event, not only controlled inflation but also moderated the debt crisis.

Related Literature Our project builds on insights from two distinct literatures on emerging market business cycle: the work on New Keynesian monetary policy in small open economies, following Gali and Monacelli (2005), and the literature on fundamental sovereign default risk, following Eaton and Gersovitz (1981).

We follow the quantitative sovereign default models of Aguiar and Gopinath (2006), Arellano (2008), with long term debt as in Hatchondo and Martinez (2009) and Chatterjee and

Eyigungor (2012).[3]. Similarly, our domestic monetary environment is close to the reference model of Gali and Monacelli (2005) and abstracts from the many extensions considered in the (medium-scale) open economy DSGE literature, e.g. Christiano et al. (2011). One methodological difference from such projects is that we use global methods rather than local approximations around the steady state. Furthermore, we focus on a simple interest rate rule to capture features of the inflation targeting regime, and do not address optimal monetary policy, along the lines of Schmitt-Grohé and Uribe (2007) or Corsetti et al. (2010).

The literature on sovereign default recently turned to questions raised by nominal rigidities and the currency denomination of debt. Na et al. (2018) emphasize downward rigidity of nominal wages and the incentives it creates for exchange rate management together with the monetary authority's inability to pursue such policies in the presence of an exchange rate peg. In their setting inflation is desirable in that, in the presence of a rigid nominal wage, adjustments in the price level can return the real wage to its efficient level. Our projects differs in two main ways: first, we consider price-setting frictions as opposed to nominal wage rigidity and, second, we model an inflation targeting monetary authority, via an interest rate rule that calls for monetary tightening in the face of rising inflation. Bianchi et al. (2018) and Bianchi and Mondragon (2018) study environments similar to Na et al. (2018) and focus on the public spending dimension of fiscal policy and the incidence of self-fulfilling debt crises. In these environments additional spending stimulates aggregate demand and lowers unemployment, but at the cost of worsened terms of borrowing. The added vulnerabilities from wage rigidities also increase the likelihood of self-fulfilling crises. We also address the fiscal authority's role in determining demand not directly through public spending but rather via international borrowing, in particular as a response to domestic monetary policy.

Hur et al. (2018) and Sunder-Plassmann (2018) study the interaction of inflation with defaultable debt denominated in local currency. The former considers exogenous inflation, for given covariance structures with fundamentals, while the latter builds on a cash-and-credit model with a constant money supply. Nuno and Thomas (2018) build a continuous time model with also local currency debt and a discretionary choice of inflation. In contrast with these papers, we emphasize the joint dynamics of endogenous inflation and country risk, under rule-based monetary policy, and focus on the structure of domestic economy as constraints facing fiscal policy-making.

A large literature, following Calvo (1988), studies whether monetary policy can alleviate self-fulfilling debt crises with defaultable debt that is denominated in local currency. Aguiar et al. (2013) study trade-offs raised by monetary policy credibility in a tractable, continuous time model of self-fulfilling default. Concerning the multiplicity of equilibria and the role inflation can play in selecting among them, Corsetti and Dedola (2016) focus on unconventional monetary policy while Bacchetta et al. (2018) embed New Keynesian features in the "slow moving debt crisis" model of Lorenzoni and Werning (2013) to quantify the role of

[3] We abstract from the recent extensions including maturity choice (Arellano and Ramanarayanan (2012), Hatchondo et al. (2016)), taxation and government spending (Cuadra et al. (2010)), and debt restructuring and renegotiation (Yue (2010), Pitchford and Wright (2012), Asonuma and Joo (2019))

inflation for crisis prevention.

Finally, our model's implications for the terms of trade, nominal and real exchange rate, and centralized borrowing raise a natural comparison with the work on capital controls and exchange rates in small open economies, such as Farhi and Werning (2012), Devereux et al. (2019) and more recently Fanelli (2017).

2 Model

We consider a small open economy which is composed of households, final good producers, intermediate goods firms, a monetary authority, and a fiscal government. There are three types of goods: imported, intermediate, and final. The final good is produced using all varieties of differentiated intermediate goods. Each variety is produced with labor. The final good is demanded by both domestic and foreign households.

Foreign demand for domestic goods (export demand) is given by

$$X_t = \left(\frac{P_t^d}{\varepsilon_t P_t^*}\right)^{-\rho} \xi,$$

where P_t^* is the price of foreign goods in foreign currency, ξ is the level of overall foreign demand and ρ is the trade elasticity. P_t^d is the price of domestic goods in local currency and ε_t is the nominal exchange rate, with an increase in ε_t being a depreciation of the home currency. We assume that the law of one price holds so we can write the price of the foreign good in local currency as

$$P_t^f = \varepsilon_t P_t^*.$$

The terms of trade e_t equal to

$$e_t = \frac{P_t^f}{P_t^d} = \frac{\varepsilon_t P_t^*}{P_t^d}. \tag{1}$$

The foreign demand for domestic goods is a function of the terms of trade

$$X_t = e_t^{\rho} \xi. \tag{2}$$

We normalize the foreign price P_t^* and the level of overall foreign demand ξ to one in all periods. Note that this implies zero inflation abroad.

2.1 Households

Households have preferences over consumption of domestic C_t and foreign goods C_t^f, as well as labor N_t. Their preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t,\ C_t^f, N_t),$$

where the per-period utility function is given by

$$u(C_t,\ C_t^f, N_t) = \log\left[H(C_t, C_t^f)\right] - \frac{N_t^{1+1/\zeta}}{1+1/\zeta}.$$

where $H\left(C_t, C_t^f\right)$ is the CES composite

$$H(C_t, C_t^f) = \left(\theta C_t^{\frac{\rho-1}{\rho}} + (1-\theta)(C_t^f)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}.$$

θ controls the share of imports in consumption while ρ is again the trade elasticity.

Taking prices as given, the households choose consumption, labor supply, and domestic bonds B_t^d holdings, denominated in local currency. Households own intermediate goods firms and receive their profits Ψ_t. They also earn labor income and receive government transfers T_t. Their budget constraint is given by

$$P_t^d C_t + (1+\tau_f) P_t^f C_t^f + q_t^d B_{t+1}^d \leq W_t N_t + B_t^d + \Psi_t + T_t$$

where q_t^d is the nominal prices of domestic discount bonds, and τ_f is a constant consumption tax that households pay on imports. It is convenient to write the budget constraint in real terms, in domestic good units, deflating by the price index P_t^d

$$C_t + (1+\tau_f) e_t C_t^f + q_t^d b_{t+1}^d \leq w_t N_t + \frac{b_t^d}{\pi_t} + \psi_t + t_t. \tag{3}$$

where real domestic bonds are $b_{t+1} = B_{t+1}^d / P_t^d$, the real wage is $w_t = W_t / P_t^d$, real profits and transfers are $\psi_t = \Psi_t / P_t^d$, $t_t = T_t / P_t^d$, as the gross domestic inflation is $\pi_t = P_t^d / P_{t-1}^d$. We can characterize the representative consumer's choices with the following optimality conditions

$$-\frac{u_{N,t}}{u_{c,t}} = \frac{W_t}{P_t^d} = w_t, \tag{4}$$

$$\frac{u_{c^f,t}}{u_{c,t}} = (1+\tau_f) e_t, \tag{5}$$

$$q_t^d = \beta \mathbb{E}_t \left[\frac{u_{c,t+1}}{u_{c,t}} \frac{1}{\pi_{t+1}}\right]. \tag{6}$$

The domestic nominal interest rate is the yield of the discount bond price $i_t \equiv 1/q_t^d$.

2.2 Final Goods Producers

The final good is produced using a measure of differentiated varieties, intermediate goods y_{it}, $i \in [0,1]$ under perfect competition,

$$Y_t = \left[\int_0^1 y_t(i)^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}. \tag{7}$$

where η is the elasticity of substitution between intermediate goods. Let the prices of intermediate goods be $\{p_t(i)\}$. The profit maximization problem of the final good producer is

$$\max P_t^d \left[\int_0^1 y_t(i)^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}} - \int_0^1 p_t(i) y_t(i) di.$$

inducing a demand function

$$y_t(i) = \left(\frac{p_t(i)}{P_t^d} \right)^{-\eta} Y_t, \tag{8}$$

and an domestic price index

$$P_t^d = \left[\int_0^1 p_t(i)^{1-\eta} di \right]^{\frac{1}{1-\eta}}. \tag{9}$$

2.3 Intermediate Goods Producers

Each differentiated intermediate good is produced with labor n_{it}, using a constant returns to scale production function, subject to aggregate productivity shocks z_t

$$y_{it} = z_t n_{it}. \tag{10}$$

Intermediate goods firms are monopolistically competitive and set the prices for their products taking as given the demand system (8). These firms, however, face price-setting frictions in that they have to pay a quadratic adjustment cost when they increase their prices relative to the target inflation rate $\overline{\pi}$, as in Rotemberg (1982). Taking as given the wage rate W_t and the final good price P_t^d, an intermediate firm i chooses labor and its price to maximize the present discounted value of profits

$$\max_{\{p_{it}, n_{it}\}} \mathbb{E}_0 \sum_t Q_{t,0} \left\{ p_{it} y_{it} - (1-\tau) W_t n_{it} - \frac{\varphi}{2} \left(\frac{p_{it}}{p_{it-1}} - \overline{\pi} \right)^2 P_t^d Y_t \right\}$$

subject to the production function, where $Q_{t,0}$ is the stochastic discount factor of households, denominated in units of domestic goods, and τ is a labor subsidy. (This subsidy is assumed constant, a fiscal policy designed to alleviate inefficiencies induced by market power, standard in the New Keynesian literature.)

Using the households' stochastic discount factor and the production function this problem is

$$\max_{\{p_{it}\}} \mathbb{E}_0 \sum_t \beta^t \frac{u_{c,t}}{u_{c,0}} \frac{P_0^d}{P_t^d} \left\{ p_{it} y_{it} - \Omega_t y_{it} - \frac{\varphi}{2} \left(\frac{p_{i,t}}{p_{i,t-1}} - \overline{\pi} \right)^2 P_t^d Y_t \right\}$$

where $\Omega_t \equiv \frac{(1-\tau)W_t}{z_t}$ is the unit cost. The first order condition for each firm, after imposing symmetry across all firms ($p_{it} = P_t^d$), results in

$$\frac{\Omega_t}{P_t^d} = \frac{\eta - 1}{\eta} + \frac{1}{\eta} \varphi \left(\pi_t - \overline{\pi} \right) \pi_t - \frac{1}{\eta} \mathbb{E}_t \left[\beta \frac{u_{c,t+1}}{u_{c,t}} \frac{Y_{t+1}}{Y_t} \varphi \left(\pi_{t+1} - \overline{\pi} \right) \pi_{t+1} \right]. \tag{11}$$

This equation is a standard New Keynesian Philips Curve (NKPC) that relates inflation to a measure of contemporaneous unit cost and expected inflation.

2.4 The Monetary Authority

The monetary authority conducts policy using a nominal interest rates rule. The nominal rates i_t is set based on the level of inflation relative to target $\pi_t / \overline{\pi}$, possibly subject to monetary shocks m_t

$$i_t = \bar{i} \left(\frac{\pi_t}{\overline{\pi}} \right)^{\alpha_P} m_t. \tag{12}$$

where the intercept $\bar{i}$ satisfies the steady state condition $\bar{i} = \overline{\pi} / \beta$.

2.5 Government and External Debt

The fiscal authority, the government engages in international borrowing using long-term bonds denominated in foreign currency. To keep long-term debt tractable, we consider random maturity bonds as in Hatchondo and Martinez (2009). The bond is a perpetuity that specifies a price q_t and a quantity ℓ_t such that government receives $q_t \ell_t$ units of foreign goods in period t. The following period a fraction δ of the debt matures and, conditional on not defaulting, the government's debt is the sum of the outstanding debt and the new issuance ℓ_t such that $B_{t+1} = (1 - \delta) B_t + \ell_t$. Each unit of debt calls for a payment of $r^* + \delta$ every period. (We normalize the debt service payment of the bond to $r^* + \delta$ so that the default-free bond price for this instrument equals 1.)

As in standard New Keynesian models, we let the fiscal government subsidize employment and foreign consumption at a time-invariant rates τ and τ_f, to correct the markup in goods markets and allow for a static optimal tariff on exports in steady state. The government transfers to households T_t, the net receipts from its operations. Letting B_t denote the outstanding foreign currency debt of the government, the budget constraint in local currency is

$$T_t + \tau W_t N_t = \varepsilon_t \left[q_t (B_{t+1} - (1 - \delta) B_t) - (r^* + \delta) B_t \right] + \tau_f P_t^f C_t^f \tag{13}$$

where the net capital inflow from debt operations is multiplied by the nominal exchange rate ε_t to convert it to domestic currency. Using equation (1), the government budget constraint in terms of domestic goods is

$$t_t + \tau w_t N_t = e_t[q_t(B_{t+1} - (1-\delta)B_t) - (r^* + \delta)B_t] + \tau_f e_t C_t^f \tag{14}$$

Every period the government experiences an enforcement shock ν_t and decides whether to default D_t on its outstanding debt. Whenever it chooses not to default it can pick the level of borrowing B_{t+1}. Default has the benefit that it eliminates all the debt obligations, but it is costly in terms of utility, productivity, and financial market access. If the government defaults, $D_t = 1$, the economy suffers a one time utility cost ν_t and a reduction in productivity to $z_t^d(z_t) \leq z_t$. In addition, a defaulting country is excluded from international financial markets for a random length of time. With probability ζ the economy re-enters financial markets with zero debt obligations.

The government's objective is to maximize the present discounted value of the flow utility derived from consumption and labor by the representative household, $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_g^t u(C_t, C_t^f, N_t)$. The government's discount factor β_g can differ from the households' β. The government borrows from competitive international lenders that discount the future at a foreign currency rate r^*. The bond price is such that they break even in expectation, thus receiving compensation for any expected losses from default

$$q_t = \frac{1}{1+r^*}\mathbb{E}_t\left[(1-D_{t+1})((r^*+\delta) + (1-\delta)q_{t+1})\right]. \tag{15}$$

In states where the government does not default, $D_{t+1} = 0$, each unit of the discount bond makes a payment $r^* + \delta$ and the fraction that does not mature, $1-\delta$, has market value $(1-\delta)q_{t+1}$. If the country does default in a state next period, the associated payoff for lenders is zero.

3 Equilibrium

We now describe the equilibrium of this economy. We consider a Markov equilibrium where the government takes into account that its default and borrowing policies affect the allocations of the private equilibrium and the monetary authority's response. In the beginning of the period, the aggregate state of this economy include the productivity, monetary, and enforcement shocks $s = \{z, m, v\}$ as well as the government debt B. The government chooses its policies, whether to default D and how much to borrowing B'. The private and monetary equilibrium depend on both the state $\{s, B\}$ and on the government choices because they affect government transfers $t(S)$. Let $S = \{s, B, D, B'\}$ be end of the period state relevant for the private equilibrium.

Definition 1. *Private and Monetary Equilibrium. Given state $\{S\}$, the government policy functions for default $D'(s', B') = H_D(s', B')$, borrowing $B''(s', B') = H_B(s', B')$, and transfer function $t(S)$ consistent with the government budget constraint, the symmetric private and monetary equilibrium consists of*

- *Households policies for domestic goods consumption $C(S)$, foreign goods consumption $C^f(S)$, labor $N(S)$, and domestic debt $B^d(S)$,*
- *Intermediate and final goods firms policies for labor $n(S)$, prices $p^d(S)$, and final domestic goods output $Y(S)$ and exports $X(S)$,*
- *The wage rate $w(S)$, domestic nominal interest rate $i(S)$, aggregate domestic price $P^d(S)$, domestic inflation $\pi^d(S)$, and the terms of trade $e(S)$*

such that: (i) *the policies for households satisfy their budget constraint (3) and optimality conditions (4), (5), (6);* (ii) *the policies of intermediate and final goods firms satisfy their optimization problem (7), (8), (10), and (11);* (iii) *export demand (2) is satisfied* (iv) *nominal interest rate satisfies the monetary authority interest rate rule (12); and* (v) *labor, domestic goods, and domestic bond markets clear, and the balance-of-payment condition is satisfied.*

The labor market clears, so that labor demanded by firms equal labor supplied by households $n = N$. Domestic bonds are in zero net supply in the economy, reflected in the market clearing condition $B^d = 0$. The resource constraint for domestic goods requires that domestic final good output equal domestic consumption and exports net of the adjustment costs

$$C(S) + X(S) + \frac{\phi}{2}(\pi - \overline{\pi})^2 Y(S) = Y(S) \tag{16}$$

where aggregate output $Y(S) = z\, N(S)$.

The balance-of-payments condition requires that net export equals the net capital outflow which here equals to the government transfer, net of the labor subsidy

$$X(S) - e(S)C^f(S)(1 + \tau_f) = t(S) - \tau w(S)N(S). \tag{17}$$

3.1 Government Recursive Formulation

We now describe the recursive problem of the government, which borrows in international financial markets and can default. The government chooses its policies internalizing that they affect the private and monetary equilibrium.

The government trades long-term discount bonds denominated in foreign currency with international lenders and can default on its debt. The government starts with debt B and decides on default D and new borrowing B' which carries price $q(s, B')$. The government internalizes that its choice of borrowing and default alter the private equilibrium. The bond price is an endogenous function that depends on the amount of borrowing B' and the shocks s, in a way that compensates lenders for default risk. These risk-neutral lenders discount the future at the international interest rate r^*. The break-even condition for them imply that the bond price schedule satisfies

$$q(s, B') = \frac{1}{1+r^*}\mathbb{E}_{s'|s}\left[(1 - H_D(s', B'))(r^* + \delta + (1-\delta)q(s, H_B(s', B')))\right] \tag{18}$$

where $H_D(s', B')$ and $H_B(s', B')$ are the default and borrowing policy function of the government.

As is standard in New Keynesian models we set that labor subsidy $(1-\tau) = \frac{\eta-1}{\eta}$ to offset the market power of firms in the steady state and set $(1+\tau_f) = \frac{\rho}{\rho-1}$ to be equal to the static optimal tariff.[4]

By consolidating the equilibrium conditions and the government budget constraint, the private and monetary allocations can be summarized with the decision rules for domestic and foreign consumption $\{C(S), C^f(S)\}$, labor $N(S)$, inflation $\pi(S)$, nominal interest rate $i(S)$, and terms of trade $e(S)$, which satisfy the following system of dynamic equations

$$C(S) + e(S)^{\rho}\xi = \left[1 - \frac{\varphi}{2}\left(\pi(S) - \overline{\pi}\right)^2\right] zN(S) \tag{19}$$

$$e(S)^{\rho}\xi = e(S)[C^f(S) + (r^* + \delta)B - q(s, B')(B' - (1-\delta)B)] \tag{20}$$

$$\frac{u_{c^f}(S)}{u_c(S)} = \frac{\rho}{\rho - 1}e(S) \tag{21}$$

$$u_c(S) = \beta\, i(S) M(s, B') \tag{22}$$

$$i(S) = \bar{i}\left(\frac{\pi(S)}{\overline{\pi}}\right)^{\alpha_p} m \quad \text{with} \quad \bar{i} = \overline{\pi}/\beta \tag{23}$$

$$\frac{1}{z}\frac{u_n(S)}{u_c(S)} = 1 + \frac{1}{\eta - 1}\varphi\left(\pi(S) - \overline{\pi}\right)\pi(S) - \frac{1}{u_c(S) zN(S)}F(s, B') \tag{24}$$

where $q(s, B')$ satisfies (18) and the functions $M(s, B')$ and $F(s, B')$ are the expectations in the

[4]By setting this tariff, we neutralize the potential incentive of the government to use debt to exert market power with respect to the downward-sloping demand for the country's exports.

households' Euler condition and the firms' pricing condition (NKPC) respectively, given by

$$M(s, B') = \mathbb{E}_{s'|s} \frac{u_c(S')}{\pi(S')} \tag{25}$$

$$F(s, B') = \frac{\beta}{\eta - 1} \mathbb{E}_{s'|s} \left[z' N(S') u_c(S') \varphi \left(\pi(S') - \overline{\pi} \right) \pi(S') \right] \tag{26}$$

where the future state is denoted by $S' = (s', H_D(s', B'), H_B(s', B'))$.

These equilibrium conditions are analogous to those arising from the standard New Keynesian small open economy in Gali and Monacelli (2005). The difference in our model is that the government understands that its choice of borrowing B' and default D, elements of S, affect the equilibrium. The equilibrium depends on government choices because current *and* future allocations and prices, as characterized by the system of equations (19) to (24), depend on B' and D.

The government's choices also determine next period's state variables, which means that future allocations and prices also depend on current government's choices. These future effects are encoded in the functions $q(s, B')$, $M(s, B')$, and $F(s, B')$, which are the bond price function, the households' expected marginal utility function, and the firms' expected inflation function, respectively. These functions are the marginal changes associated with a change in the B' choice taking as given future governments policies $H_D(s', B')$ and $H_B(s', B')$.

We can now set up the the recursive problem of the government, following the quantitative sovereign default literature. The government can choose to default in any period. Let $V(s, B)$ be the value with the option to default. After default, the debt B is eliminated, productivity is reduced to $z^d(z)$, and the government suffers the default cost ν. The value of the option to default is then

$$V(s, B) = \max_{D \in \{0,1\}} \left\{ (1 - D) W(s, B) + D \left[W^d(z^d, m) - \nu \right] \right\} \tag{27}$$

where $D = 1$ in default and 0 otherwise, $W(s, B)$ is the payoff from repaying debt, and $W^d(z^d, m) - \nu$ is the payoff from defaulting. Specifically, the value of repaying is

$$W(s, B) = \max_{B'} \left\{ u(C, C^f, N) + \beta_g \mathbb{E}_{s'|s} V(s', B') \right\} \tag{28}$$

subject to the private and monetary equilibrium which is characterized by conditions (19) through (24), and the break even condition for the bond price schedule (18).

After default, with probability ξ the government is forgiven and regains access to the international financial markets with zero debt. The defaulting value W^d net of the enforcement cost is given by

$$W^d(z^d, m) = \left\{ u(C, C^f, N) + \beta_g \mathbb{E}_{s'|s} \left[\xi V(s', 0) + (1 - \xi) W^d(z^{d\prime}, m') \right] \right\} \tag{29}$$

subject to the private and monetary equilibrium characterized by conditions (19) through (24) with $B = B' = 0$ and penalized productivity $z^d(z)$.

It is convenient to write the default decision of the government as a cutoff rule based on the default cost ν. Given that default costs are i.i.d., the default decision $D(s, B)$ can be characterized by a cutoff default cost $\nu^*(z, m, B)$ at which the repayment value is equal to the default value such that

$$\nu^*(z, m, B) = W(z, m, B) - W^d(z^d, m), \tag{30}$$

and the sovereign is indifferent between the two options. Then $D(s, B) = 1$ whenever $\nu \leq \nu^*(z, m, B)$ and zero otherwise. Let Φ be the cumulative distribution of ν. The default probability is therefore given by $\Phi(\nu^*(z, m, B))$.

We now define the recursive equilibrium for the economy.

Definition 2. *Equilibrium. Given the aggregate state $\{s, B\}$ a recursive equilibrium consists of government policies for default $D(s, B)$ and borrowing $B'(s, B)$, and government value functions $V(s, B)$, $W(s, B)$, and $W^d(s, B)$ such that*

- *Taking as given future policy and value functions $H_D(s', B')$, $H_B(s', B')$, $V(s', B')$, and $W(s', B')$, government policies for default and borrowing $D(s, B)$ and $B'(s, B)$ solve the government's optimization problem.*
- *Government policies and values are consistent with the future policies and values.*

3.2 Borrowing with Default Risk and Monetary Policy

In this section we characterize the optimal borrowing decision. For tractability we restrict attention to the Cobb-Douglas case for the H consumption aggregator, corresponding to a unit elasticity of substitution. We continue to allow for an arbitrary export elasticity ρ. As described in the recursive equilibrium, the government chooses its borrowing taking into account the effect that borrowing has on the private equilibrium, both contemporaneously and in the future. We manipulate the government's problem and derive its optimality condition for borrowing to illustrate the forces at play. In this derivation we have assumed that the functions in the government problem are differentiable.[5] Optimal borrowing satisfies the following Euler equation

$$\begin{aligned} u_{C_f}\left[q + \frac{\partial q}{\partial B'}\left(B' - (1-\delta)B\right)\right](1-\tau_m) = \beta_g \mathbb{E} u'_{C_f}(r^* + \delta + (1-\delta)q')(1-\tau'_m) \\ - \beta \mathbb{E}\left[\Phi(\nu^*(z', m', b'))\frac{\partial \nu^*}{\partial B'}\right] + \gamma \frac{1}{u_C z N}\frac{\partial F(s, B')}{\partial B'} \end{aligned} \tag{31}$$

[5]We do not require this assumption for the computation of the model, nor do we employ the Euler equation derived in this section.

where we supressed the dependency of τ_m and γ on the current state and borrowing B', and Φ is the cumulative distribution of the default cost shock ν. γ is the multiplier on the NKPC condition. The left-hand side of the Euler is the marginal benefit, and the right-and side is the marginal cost of an extra unit of borrowing.

We first focus on τ_m and then explain the forces affect the government's borrowing incentives. We call τ_m the *monetary wedge*. It is given by

$$\tau_m = \left[\kappa\theta - \gamma\frac{u_N}{zu_c}\left(1 + F(s, B')\right)\right]\frac{\rho X u_c}{\rho X u_c + \theta(1-\theta)},$$

where κ is the multiplier on the domestic Euler condition (22) and γ is the multiplier on the NKPC (24). We make two remarks here. First, when there are no monetary frictions, as in a real model, $\kappa = \gamma = 0$, the monetary wedge is inactive and $\tau_m = 0$. Hence the presence of the monetary wedge is due to the price rigidities. Second, this monetary wedge is positive when consumption is too low relative to leisure. As is standard in New Keynesian models, pricing frictions lead to labor wedges that distort production away from the level that would equates that marginal product of labor to the marginal disutility of consumption relative to labor namely, $u_C \neq \frac{u_N}{z}$. In our model, this wedge impacts government borrowing. A positive $\tau_m > 0$ decreases the marginal benefit of borrowing because a higher B' tends to increase capital inflows and default risk, appreciate the exchange rate, reduce exports, and lower the demand for domestic goods, worsening the domestic monetary distortion.

The government's borrowing incentives are affected by three major forces. The first is the standard force to smooth imported consumption. It is useful to compare our model's Euler equation (31) with an undistorted Euler equation that arises in the standard Gali and Monacelli (2005) model without sovereign borrowing, under market incompleteness and long-term debt,

$$q\, u_{Cf} = \beta\mathbb{E}\left[u'_{Cf}(r^* + \delta + (1-\delta)q')\right]. \tag{32}$$

Recall that, given a level of borrowing B' and its price q, the allocations and prices in our model are exactly the same as the allocations in this reference model as both models satisfy the system of equations (19) through (24).

The second force that affects the government's borrowing incentive is the endogenous bond price schedule q and the legacy debt $(1-\delta)B$. Due to default risk, bond prices decrease with borrowing to reflect higher default risk, $\frac{\partial q}{\partial B'} \leq 0$. Also, a higher legacy debt $(1-\delta)B$ increase borrowing incentives because lower prices dilute this debt, $-\frac{\partial q}{\partial B'}(1-\delta)B \geq 0$ This time-inconsistency in debt issuances and dilution incentives have been studied in the sovereign default literature in Chatterjee and Eyigungor (2012); Hatchondo et al. (2016). Such dilution incentives is on one potential source of overborrowing in our model.

The third force works through the monetary wedge and is unique to our model, due to the presence of monetary frictions. A positive $\tau_m > 0$ decreases the marginal benefit from borrowing B' and as we explain below will be positive when monetary frictions are large. This force therefore helps discipline the incentive to overborrow of the government.

There are two additional terms affect the government's borrowing incentives in (31). One is the "default discount" $-\beta\mathbb{E}\left[\Phi(\nu^*(z',m',b'))\frac{\partial\nu^*}{\partial B'}\right]$, which reflects the net benefit of not having to service the debt in default states. The other term relates to the response of future inflation to the debt carried into next period, $\gamma\frac{1}{u_C zN}\frac{\partial F(s,B')}{\partial B'}$. In our quantitative analysis, these two terms are small.

3.3 The Perfectly Rigid Case

To illustrate how the government's borrowing and default behavior impact the monetary friction, we focus on a case with perfectly sticky prices, i.e. $\varphi \to \infty$. In this environment, firms set their prices at period 0 once for all, and the prices remain at that level permanently. We fix the nominal interest rate i to its neutral level $\bar{i}$. We exclude the productivity and monetary shocks so that the only shock is the idiosyncratic default cost ν. We continue to assume the following preference

$$u(C, C^f, N) = \theta \log C + (1-\theta)\log C^f - \chi\frac{N^{1+\mu}}{1+\mu}.$$

We also assume $\rho \geq 1$ and that equilibrium functions are continuous and differentiable. After default, international financial exclusion is permanent. The state of the private equilibrium is $S = (B, B')$, assuming the government did not default.

Let the private equilibrium schedule, under normal market access, and a debt choice for the government B' consists of $\{C(S), C^f(S), N(S), X(S), e(S)\}$, the equilibrium allocation in default is $\{C_d, C^f_d, N_d, X_d, e_d\}$. Let the equilibrium outcome of private economy conditional on not defaulting be $\{H_C(B), H_{C^f}(B), H_N(B), H_X(B), H_e(B)\}$. Under consistency conditions, for any variable $y \in \{C, C^f, N, X, e\}$ without default, $\hat{y}(B) = H_y(B, H_B(B))$ where $H_B(B)$ is the equilibrium debt choice function of the government. In addition, let $\Phi(B)$ be the government's equilibrium default probability function. Let Φ_ν be the cumulative distribution function of ν, and ν^* is the default cutoff which solves $W(B) = W^d - \nu^*$. Hence the default probability function satisfies $\Phi(B) = \Phi_\nu(\nu^*(B))$.

The private equilibrium with market access satisfies the following conditions

$$C(S) + e(S)^\rho \xi = zN(S) \tag{33}$$

$$e(S)^\rho \xi = e(S)[C^f_t(S) + (r^* + \delta)B - q(B')(B' - (1-\delta)B)] \tag{34}$$

$$\frac{u_{c^f}(S)}{u_c(S)} = \frac{\rho}{\rho - 1}e(S) \tag{35}$$

$$u_c(S) = \beta\, i\left[(1 - \Phi(B'))u_c(B') + \Phi(B')u_{c_d}\right] \tag{36}$$

$$q(B') = \frac{1}{1+r^*}\left[(1 - \Phi(B'))(r^* + \delta + (1-\delta)q(H_B(B')))\right] \tag{37}$$

$$1 = \mathbb{E}_0 \left\{ \sum_t \left[\frac{\beta^t u_{c,t} z_t N_t}{\mathbb{E}_0 \sum_s \beta^s u_{c,s} z_s N_s} \right] \frac{u_{N,t}}{z_t u_{c,t}} \right\}, \tag{38}$$

where equation (33)-(36) are the same as in the benchmark case, with $\pi_t = \overline{\pi}$ for any t outside default. The perfectly rigid equilibrium depends on the constant price level $\overline{P}$, which we assume firms set at time 0. The last equation (38) reflects this optimal choice of pricing, given by the life-time average of unit costs, weighed by output levels. Note that in comparison with the benchmark model with costly price adjustment, the present model does not feature a NKPC condition.

The allocation in default satisfy the budget constraint $C_d + e_d^\rho \xi = z_d N_d$, balanced trade $e_d^\rho \xi = e_d C_d^f$, and the relative consumption condition $u_{c_d^f} / u_{c_d} = \frac{\rho}{\rho - 1} e_d$.

The monetary wedge under the perfectly rigid case is

$$\tau_m = \left(1 - \frac{1}{z u_C / u_N} \right) \frac{\rho X u_C}{\rho X u_C + (1 - \theta)} \tag{39}$$

where the labor wedge $z u_C / u_N$ reflects the model distorted labor markets. The monetary wedge is therefore positively associated with the labor wedge.

Proposition 1. *Suppose $\partial[q(B')B']/\partial B' \geq 0$ and $\partial H_C(B)/\partial B \leq 0$. An arbitrary increase in B' leads to an increase in the default risk $\Phi(B')$, a decline in domestic consumption, labor, and export, and an appreciation, i.e. $\partial C(B, B')/\partial B' \leq 0$, $\partial N(B, B')/\partial B' \leq 0$, $\partial X(B, B')/\partial B' \leq 0$, $\partial e(B, B')/\partial B' \leq 0$. The labor wedge, $lw(B, B') = z u_C / u_N$ increases with B' as well, $\partial lw / \partial B' \geq 0$.*

This result shows that increasing borrowing increases monetary frictions, as measured by the labor wedge, for two reasons. First, a higher B' increases default risk $\Phi(B')$ which lowers expected domestic consumption next period because consumption in default is low. A lower expected consumption lowers current domestic consumption given that the domestic interest rate i is fixed, as seen in equation (36). This depressed demand reduces domestic production. The low domestic demand due to higher default risk is one force that worsens the labor wedge. Second, high B' increases capital flows which appreciate the terms of trade e, lowering export demand and further depressing production of domestic goods. This low foreign demand for domestic goods due to high capital inflows is the second force that worsens the labor wedge.

3.4 Price Indices and Exchange Rates

It is useful to define relations between the terms of trade, exchange rates, consumer and producer price indices. Recall that the π in the model corresponds to PPI inflation, since it reflects the pricing of domestic goods alone, while e is the relative price of foreign goods, the terms of trade. We can derive the consumer price index as the price of bundle of domestic and imported consumption goods,

$$P^{\text{CPI}} = \left[\theta^\rho {P^d}^{1-\rho} + (1-\theta)^\rho {P^f}^{1-\rho}\right]^{\frac{1}{1-\rho}} = P^d \left[\theta^\rho + (1-\theta)^\rho e^{1-\rho}\right]^{\frac{1}{1-\rho}}$$

and the resulting CPI inflation

$$\pi^{\text{CPI}} = \frac{P^{\text{CPI}}}{P^{\text{CPI}}_{-1}} = \pi \frac{\left[\theta^\rho + (1-\theta)^\rho e^{1-\rho}\right]^{\frac{1}{1-\rho}}}{\left[\theta^\rho + (1-\theta)^\rho e_{-1}^{1-\rho}\right]^{\frac{1}{1-\rho}}}.$$

We can solve for the CPI-based real exchange rate and its rate of depreciation

$$\pi_{e,\text{CPI}} = \frac{e}{e_{-1}} \frac{\left[\theta^\rho + (1-\theta)^\rho e_{-1}^{1-\rho}\right]^{\frac{1}{1-\rho}}}{\left[\theta^\rho + (1-\theta)^\rho e^{1-\rho}\right]^{\frac{1}{1-\rho}}}$$

as well as the rate of depreciation of the nominal exchange rate

$$\frac{\varepsilon}{\varepsilon_{-1}} = \frac{e}{e_{-1}} \frac{P^d}{P^d_{-1}} = \frac{e}{e_{-1}} \pi.$$

Note that CPI inflation and the rates of depreciation (real and nominal, respectively) require knowledge of previous period's terms of trade (e_{-1}). Finally, taking the limit $\rho \to 1$ for the consumption CES aggregator we can recover the Cobb-Douglas case, with $\pi^{\text{CPI}} = \pi\,(e/e_{-1})^{1-\theta}$ and $\pi_{e,\text{CPI}} = (e/e_{-1})^{\theta}$.

4 Quantitative Analysis

We now describe the parameterization of the model, discuss policy rules, impulse responses, and compare the model implications to the data. Finally, we use the model to simulate a counterfactual loose monetary policy during the 2015 recession in Brazil.

4.1 Inflation, Nominal Rates, Output, and Spreads in the Data

Table 1 reports key statistics on the joint behavior of inflation, spreads, output, and domestic nominal rates, for a set of emerging markets. We start the sample in 2004, by which point all countries considered had adopted inflation targeting for their monetary policy. Data is quarterly. Inflation is CPI-based and computed related to 4 quarters prior. The spreads are EMBI-based, the difference in yields between foreign-currency government bonds of these emerging markets and a U.S. government bond of similar maturity. Domestic nominal rates are short-term rates in local-currency from either inter-bank markets or government instruments, the shortest maturity available. Output is 4 quarter difference in log GDP. We highlight several salient features of the data which will inform our quantitative work.

	Means		Correlation with Spread		
	Inflation	Govt Spread	Inflation	Domestic Rate	Output
Brazil	5.9	2.6	59	59	-62
Chile	3.0	1.4	30	39	-49
Colombia	5.2	3.2	74	76	-60
Indonesia	6.6	2.8	17	75	-62
Korea	2.6	1.1	44	74	-30
Mexico	4.3	2.3	48	27	-54
Peru	2.8	3.0	50	55	-33
Philippines	3.9	2.9	17	82	-26
Poland	3.0	1.7	59	52	-11
South Africa	5.8	1.9	54	20	-49
Mean	**4.4**	**2.4**	**45**	**58**	**-38**

Table 1: Emerging Market Inflation Targeters, Key Statistics

Inflation is low for these inflation targeting emerging markets. These single-digit inflation patterns contrast sharply with the historical experience of these countries, which have featured several episodes of hyperinflation. In our view, these emerging markets have conquered inflation. The table also shows average EMBI spreads for these emerging market, which continue to be sizable.

We also report correlations of spreads with inflation, domestic rates, and output. As documented in many studies, spreads are negatively correlated with output for this sample with an average correlation of -38%. Correlations of spreads with nominal rates are strongly positive, on average 58%. Note that these two rates are in different currencies and hence reflect positive comovements between inflation and default risk. The correlation between spreads and inflation is positive, with sample average of 45%.

4.2 Parameterization

The model features 3 shocks: productivity z, monetary m, and enforcement ν. We assume that productivity follow an AR(1) process $\log z_t = \rho_z \log z_{t-1} + \sigma_z \varepsilon_t$ with $\varepsilon_t \sim \mathcal{N}(0,1)$. Following Chatterjee and Eyigungor (2012), we assume that while in default productivity suffers a convex penalty $z^d(z) = z - \max\left\{0, \lambda_0 z + \lambda_1 z^2\right\}$ with $\lambda_0 < 0, \lambda_1 > 0$. For the calibration and simulation of the model we abstract from monetary shocks, but we incorporate unexpected (zero probability) monetary shocks to analyze their associated impulse response functions and to construct counterfactual monetary policy for our Brazil event study. For numerical stability purposes we augment the model with taste shocks, in the discrete choice tradition, following Dvorkin et al. (2018) and Gordon (2018). Appendix A details the structure of these shocks, their numerical properties, and their role in robust convergence of models with long-term debt, such as ours. We slightly perturb the borrowing B' choice as well as the default decision. The shocks to the default-vs-repayment decision map into the model's enforcement shock ν. In the model with taste shocks, the default decision becomes

$$\begin{aligned} V(s,B) &= \mathbb{E}_{\epsilon_{\text{Repay}},\epsilon_{\text{Default}}} \max\left\{W(s,B) + \varrho_D\,\epsilon_{\text{Repay}},\ W_d(s) + \varrho_D\,\epsilon_{\text{Default}}\right\} \\ &= \mathbb{E}_{\epsilon_{\text{Repay}},\epsilon_{\text{Default}}} \max\{W(s,B),\ W_d(s) + \varrho_D\,\underbrace{(\epsilon_{\text{Default}} - \epsilon_{\text{Repay}})}_{\nu}\}. \end{aligned}$$

Here the ϵ terms are iid Gumbel (Extreme Value Type I) and their difference ν follows the Logistic distribution. The parameter ϱ_D controls the relative importance of the enforcement shock for the default decision.

Assigned Parameters		
Share domestic in consumption	$\theta = 0.62$	Import share of Brazil
Frisch elasticity	$\mu = 0.33^{-1}$	Gali and Monacelli (2005)
Persistence of productivity	$\rho_z = 0.9$	Literature
Trade elasticity	$\rho = 5$	Devereux et al. (2019)
Export demand level	$\xi = 1$	Normalization
Varieties elasticity	$\eta = 6$	Literature, 20% markup
Interest rate rule intercept	$\bar{i} = \overline{\pi}/\beta$	Equilibrium condition
International rate	$r^* = 0.5\%$	US Treasury yields
Market reentry probability	$\zeta = 4.17\%$	Average 6 year exclusion
Price adjustment cost	$\varphi = 58$	Calvo⇔Rotemberg conversion
Parameters from Moment Matching		*Brazilian Data, 2004–2017*
Private discount factor	$\beta = 0.9866$	Mean nominal rate
Government discount factor	$\beta_g = 0.9766$	Consumption volatility
Inflation target	$\overline{\pi} = 1.015$	Mean inflation
Interest rate rule	$\alpha = 1.4$	Inflation volatility
Std of productivity shock	$\sigma_z = 0.95\%$	Output volatility
Productivity in default	$\lambda_0 = -0.17$	Government spread
	$\lambda_1 = 0.19$	Spread volatility
Default taste shock	$\varrho_D = 1e^{-4}$	Corr output & spread

Table 2: Parameter Values

We calibrate our model to quarterly Brazilian data from 2004 to 2017. Table 2 presents all the parameters values with the source or targeted moments while Table 3 reports the fit. There are two sets of parameters. For the first, we assign their values directly, by relying on one-to-one mappings with the data or reference values in the literature. The second set is unique to Brazil and we choose them jointly to match data moments. The first set of parameters include the inverse of the Frisch elasticity μ, the share of domestic goods in

	Data (%)	NK-Default
Mean inflation	5.9	5.9
Mean domestic rate	11.2	11.1
Volatility of inflation	1.8	1.8
Volatility of output	1.9	1.9
Volatility of consumption	1.8	2.0
Mean spread	2.6	2.6
Volatility of spread	0.9	0.9
Output, spread correlation	−62	−60

Table 3: Model Fit

consumption θ, the trade elasticity ρ, the international interest rate r^*, varieties' elasticity and markups η, the persistence of the producivity shock ρ_z, the probability of return to financial markets after default ζ, and the Rotemberg adjustment cost φ. For the Frisch elasticity, we choose a value of 0.33 following Gali and Monacelli (2005). This is a conservative value in line with the open economy New Keynesian literature. The import share of Brazil is 15 percent, we therefore set θ to 0.6225 given the CES structure of the consumption aggregate. The trade elasticity ρ is set at 5, as in Devereux et al. (2019). This number is within the range of estimates in the trade elasticity literature. The international risk-free rate is 2%, consistent with US Treasury yields. The elasticity of substitution between varieties η is 6, standard in the literature, inducing a 20% markup. We target an average length of market exclusion of roughly 6 years. Given that we are considering a short horizon of the data and that the employment data for Brazil has many missing values, it is difficult to estimate precisely the productivity process. Instead, we set the persistence parameter ρ_z to a reference value of 0.9, comparable with most IRBC calibrations, and set the volatility of the innovation σ_z to match the unconditional standard deviation of Brazilian output. We set the Rotemberg adjustment cost using the well-known first-order equivalence between Calvo and Rotemberg pricing frictions: given our varieties' elasticity of $\eta = 6$ and a Calvo frequency of price changes of roughly once per year (once every fourth quarter), we set φ to 58.

The second group of parameters includes the discount factor of the private sector β and of the government β_g, the inflation target $\overline{\pi}$, the interest rate rule coefficient α_p, parameters related to productivity, and a parameter governing the taste shock on the default decision. We choose these jointly so that the model replicates observed moments. The volatility of productivity is the main driver of that of output. With an interest rate rule and subject to Rotemberg adjustment costs, it is costly to deviate from the inflation target $\overline{\pi}$, and thus the average inflation rate in the data pins down $\overline{\pi}$. We use α_p to match CPI inflation volatility. The productivity loss parameters λ_0 and λ_1 discipline spreads, mean and volatility, therefore we use them to match EMBI statistics. The discount factor of the private sector β pins down the average domestic real rate and, together with mean inflation $\overline{\pi}$, determines the mean domestic nominal rate $\bar{i}$. As common in the sovereign default framework, we assume that the fiscal authority is relatively impatient compared to both the private sector and international investors, $\beta_g \leq \beta \leq (1 + r^*)^{-1}$. The extra discounting of the government helps match spreads statistics but mainly contributes to replicating the volatility of aggregate consumption (domestic and imported goods).

Finally, computing our model with discrete choice methods introduces two parameters, governing the added uncertainty over the borrowing choice and default, respectively, as discussed in Appendix A. We set the parameter governing the importance of enforcement shocks ϱ_D to $1e^{-4}$, to match the correlation of spreads with output in the data. Moreover, enforcement shocks make the bond price schedule and the one-period-ahead default probability smoothing functions of the borrowing choice, ameliorating the numerical convergence properties of the model. As discussed by Chatterjee and Eyigungor (2012) convergence with long-term debt can greatly be enhanced by slight perturbations of the borrowing decision, as shocking the default option alone is often insufficient. We set the parameter governing the shock to borrowing ϱ_B to $1e^{-6}$, a value that guarantees fast, near monotone convergence for a wide range of parameter values, while at the same time keeping choice probabilities quite tight (As illustrated in Appendix A, over most of the state space, about 70% of the probability mass over B' is concentrated over 3-4 neighboring grid points).

4.3 Reference Models

To better understand the interactions between monetary frictions and default risk, we compare our findings to two reference models, one without default risk, close to the standard Gali and Monacelli (2005) model, the other one without monetary frictions, similar to the standard sovereign default setting of Arellano (2008), allowing for long-term debt as in Hatchondo and Martinez (2009).

We call the first one, without default risk, **NK-Reference**. The equilibrium of this model is characterized by conditions (19–24), an exogenous debt-elastic bond price schedule to close the model as in Schmitt-Grohé and Uribe (2003), and international borrowing without sovereign debt and default, summarized in equation (32).

The debt-elastic bond price schedule of NK-Reference is

$$q^*(B)^{-1} = 1 + r^* + \Gamma\left[\exp(B - \overline{B}) - 1\right]$$

and we set $\Gamma = 1e^{-7}$, a very loose borrowing schedule, close to the complete markets case of Gali and Monacelli (2005).

We call the second model **Default-Reference**. In this model, we shut down the price frictions and eliminate the monetary policy. The equilibrium of this model is therefore characterized by conditions (19–21), and efficient labor allocation $u_n/u_c = z$ which implies that the labor wedge is 1. In the Default-Reference model, the fiscal authority is not constrained by the presence of a domestic monetary friction that responds to its borrowing choices and therefore lacks this additional incentive to reduce debt.

We solve the NK-Reference using Dynare 4.5 for MATLAB, with a first-order log-linear approximation of the equilibrium conditions. We compute the Default-Reference model with the same global methods employed in the computation of NK-Default and documented in Appendix A. For both reference models, we keep parameter values the same as in NK-Default where applicable.

4.4 Policy Rules

Before describing the model-generated time series, we illustrate the main mechanisms by describing policy rules and key equilibrium objects. The allocation in the Private and Monetary Equilibrium depends on both the state (s, B) but also any arbitrary borrowing choice by the government (B'). We focus on behavior on the equilibrium path and discuss the resulting allocations as a function of the debt level B, fixing productivity at its median level $z = 1$, as illustrated in the panels of Figure 1.

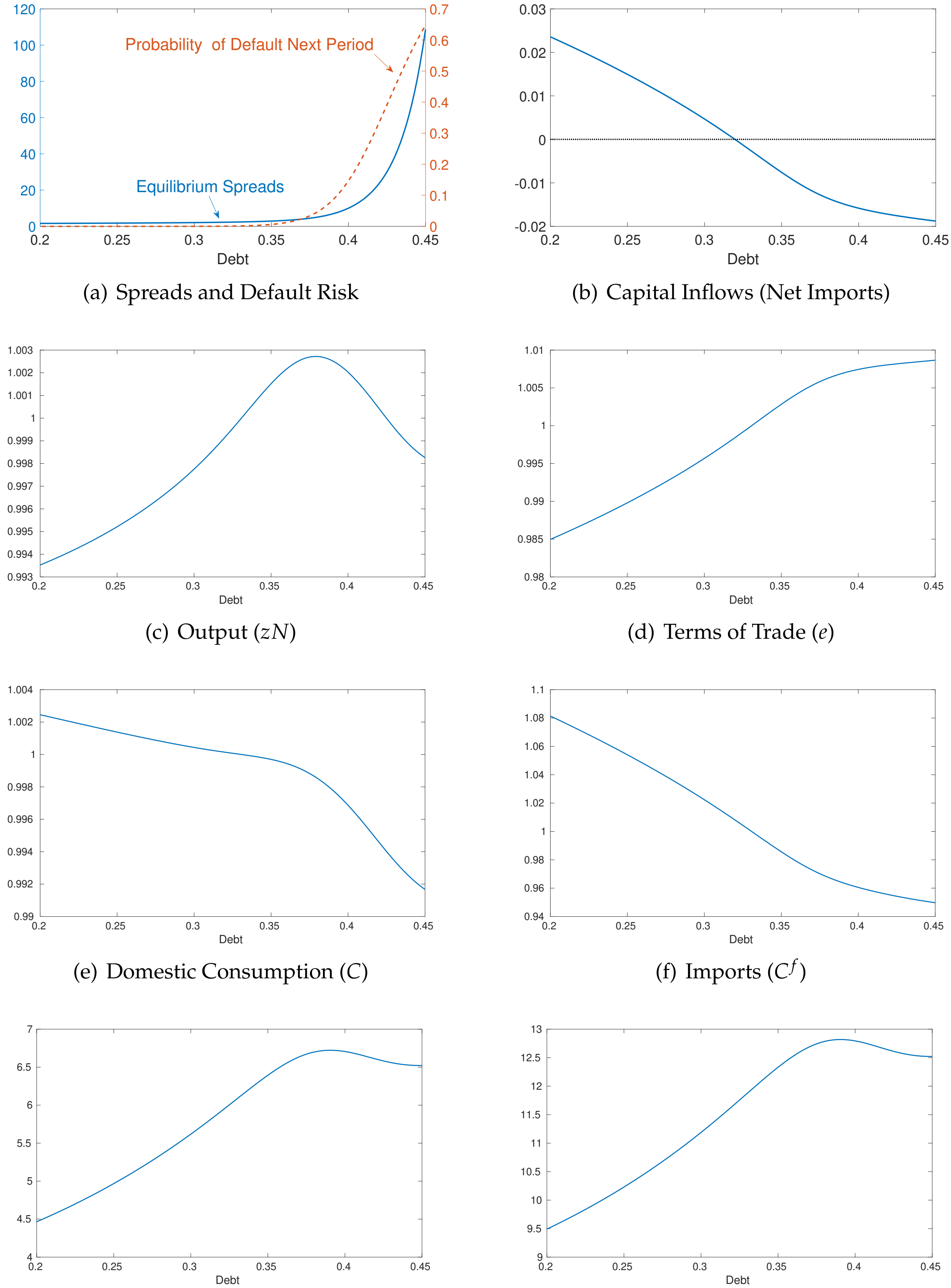
Panel (a) plots bond yield spreads paid at various debt levels (solid blue) and the one-period-ahead default probability (dashed red). We recover yield-to-maturity spreads from equilibrium bond prices using

$$\text{spread}(s, B') = (r^* + \delta)\left[\frac{1}{q(s, B')} - 1\right].$$

Throughout we report annualized values for spreads, interest rates, and inflation rates. Due to the presence of long-term debt, spreads are everywhere positive and smoothly increasing with the state. Concerning the probability of default next period, we emphasize two regions, a key to understanding the behavior of the model: a largely risk-free "Low Default Zone," for B below roughly 0.35, and a "High Default Zone," for higher levels of debt. In the High Default Zone, the probability of default sharply increases in the current debt level.

Panel (b) plots the capital inflows (net imports, expressed in foreign goods) on the equilibrium path. At low debt levels the government borrows aggressive and inflows are large and positive. As debt increases, the bond price schedule tightens and eventually capital inflows reverse. Into the High Default Zone, the country is a net exporter, unable to rollover debt in full.

Panels (c) through (h) display key variables of the Private and Monetary Equilibrium. We stress that behavior is markedly different in the Low Default Zone compared with the High Default one. With low risk, higher debt is associated with lower capital inflows (panel (b)) and a higher debt service, requiring that the country works and exports more. As domestic goods become relatively more abundant the terms of trade depreciate (panel (d)) which implements the expansion of exports. The higher level of activity stimulates inflation (panel (g)) and the monetary authority responds by tightening via higher rates (panel (h)), as prescribed by the interest rate rule. With higher debt, low inflows, and a higher domestic nominal rate,



(a) Spreads and Default Risk

(b) Capital Inflows (Net Imports)

(c) Output (zN)

(d) Terms of Trade (e)

(e) Domestic Consumption (C)

(f) Imports (C^f)

(g) Inflation (π)

(h) Nominal Interest Rate (i)

Figure 1: Policy Rules

consumption of both domestic and foreign goods decreases (panels (e) and (f) respectively).

In the High Default Zone, when debt is high and the probability of default next period is positive and rising, an additional mechanism comes into effect. In our model, during default productivity is low and the country is temporarily excluded from borrowing, so that defaults are associated with low consumption and high inflation. When the probability of such an outcome increases, households cut consumption since they expect the marginal utility of consumption next period to be high (as reflected in the RHS of the Domestic Euler equation) and the nominal interest rate stays high, to fight current inflation. The net result is a sharp reduction in consumption, enough that production of domestic goods can decrease. As discussed in the next section, the net result of these changes in consumption and activity is a more distorted economy, with a higher labor wedge, a channel absent in models without default risk.

4.5 Default Risk and Monetary Frictions

Given the behavior discussed in the previous section, we now turn to the relation between default risk and monetary distortions in the domestic economy. We focus on the labor wedge as a measure of these distortions. Figure 2 plots, as a function of debt, both the labor wedge (left axis) and the one-period-head default probability (right axis). We plot the labor wedge relative to 1. We note that in the Low Default Zone, consistent with our discussion of consumption and labor above, the labor wedge is decreasing in the level of debt. The economy needs to work more to export and service the debt due to the tight bond price schedule. As employment expands the labor wedge falls.

In the High Default Zone, the consequences of debt for the labor wedge are different: rising default risk sharply depresses domestic consumption, leading to inefficiently low demand and a substantially depressed level of activity. The labor wedge rises rapidly with the debt level. The reduction in demand due to high default risk does not lead firms to reduce inflation, as seen in panel (h). Without high current inflation, the monetary authority keeps the nominal interest rate fairly flat in this region. Inflation is this zone is not a good proxy for the monetary distortion, unlike in the Low Default Zone.

The disconnect between current inflation and the monetary distortion in the High Default Zone can be understood in terms of the New Keynesian Philips Curve, reproduced below for convenience:

$$\frac{1}{z}\frac{u_n(S)}{u_c(S)} = 1 + \frac{\varphi}{\eta - 1}\left\{(\pi(S) - \overline{\pi})\,\pi(S) - \beta\mathbb{E}_{s'|s}\left[\frac{z'\,N(S')\,u_c(S')}{z\,N(S)\,u_c(S)}\,(\pi(S') - \overline{\pi})\,\pi(S')\right]\right\}$$

Note that the LHS is the inverse of the labor wedge which will be different from 1 whenever inflation either today or next period is away from target. In the Low Default Zone, future inflation is close to target and the expectation term on the RHS is small. Then, current inflation is tightly connected to the labor wedge and when the monetary authority responds to deviations of inflation from target, they are in effect responding to the labor wedge. In this

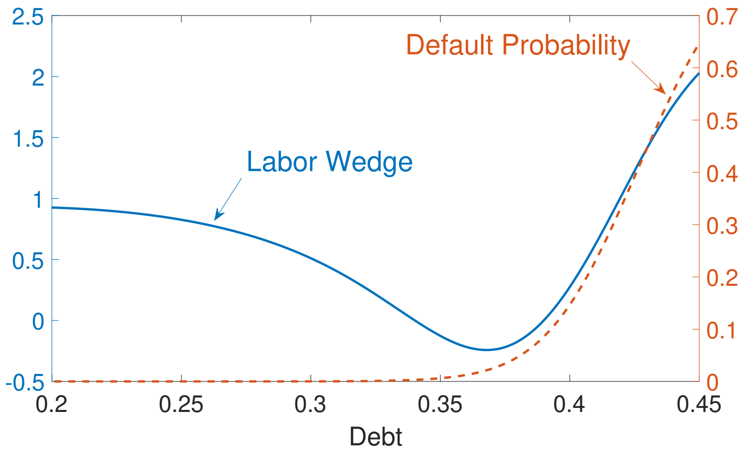
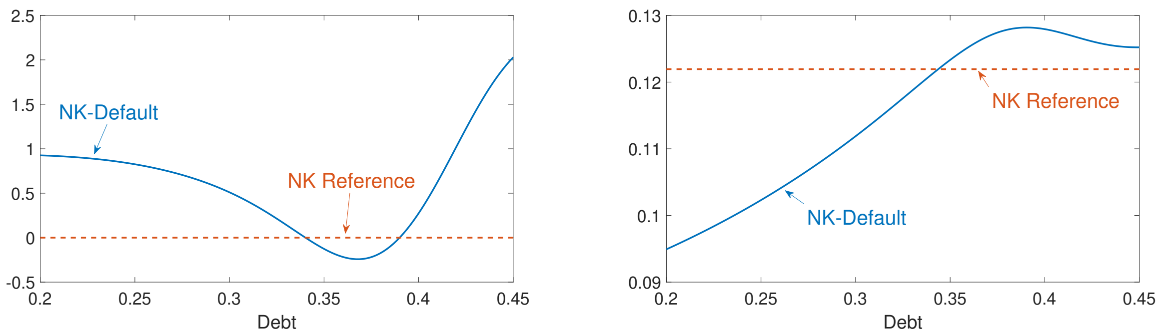


Figure 2: The Labor Wedge and the Probability of Default Next Period

zone, effectively the monetary authority is leaning against the friction. In the High Default Zone, in contrast, the expectation on the RHS is positive and reflects the low consumption and high inflation during default. These future inflation and consumption expectations create a disconnect between current inflation and the labor wedge. Firms are not lowering prices as they risk price increases in the future, if the government defaults. Unresponsive inflation implies that the monetary authority does not lean against the friction in this zone.



(a) The Labor Wedge (zu_C/u_N) (b) Nominal Interest Rate (i)

Figure 3: Debt and the Labor Wedge

Figure 3 compares the consequences of debt in the NK-Default baseline model with the NK-Reference model. Panel (a) compares labor wedges while panel (b) shows domestic nominal interest rates. In the reference model, debt does not distort the level of activity; the labor wedge is flat and zero. For our benchmark model, in the Low Default Zone capital inflows decrease in the level of debt resulting in higher export demand and a reduced labor wedge while in the High Default Risk, default risk directly depresses consumption, leading to a rapid increase in the monetary distortion. In contrast, in the NK-Reference model with very loose borrowing, inflows are nearly insensitive to debt and the model is silent on default risk. Production, inflation, and consumption therefore do not respond to debt in a sizable fashion, leaving distortion unchanged.

Panel (b) of Figure 3 compares the nominal interest rates as a function of debt in our benchmark model to the NK-Reference. In our model nominal rates respond to the level of debt, because monetary distortion vary with the level of debt. In contrast, interest rates and the monetary distortion in the NK-Reference model do not respond to debt.

4.6 Monetary Frictions Discipline Borrowing

We have argued in Sections 3.2 and 3.3 that the presence of monetary frictions disincentivizes borrowing. We illustrate this in the context of our quantitative results. We do so by comparing borrowing and spreads in our model, NK-Default, and its Real version.

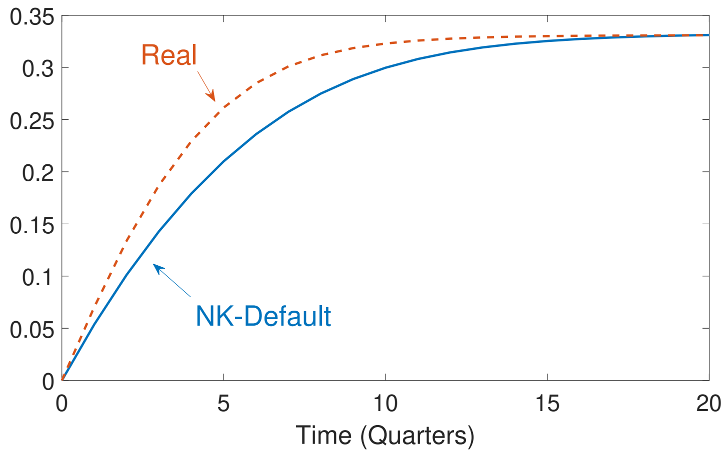
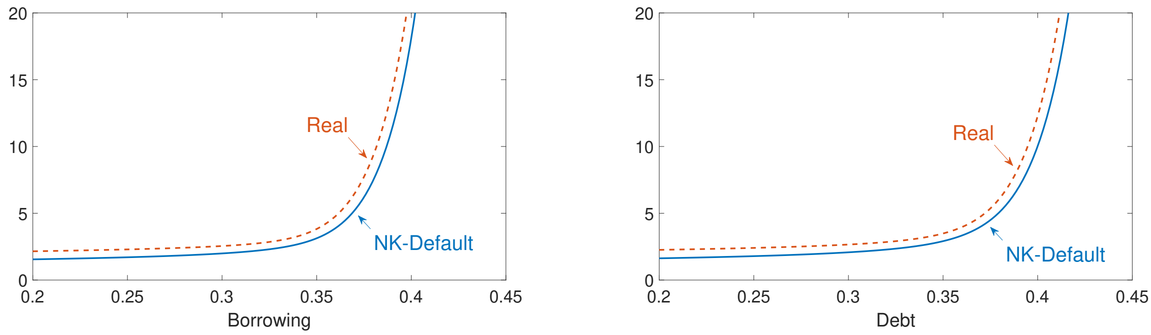


Figure 4: Debt Accumulation

Figure 4 compared the pattern of debt accumulation in the two models: both models are simulated, starting with zero debt and keeping productivity at mean throughout. The Default-Reference model accumulates debt faster, reaching close to the "steady state" level of debt roughly one year before NK-Default. Eventually the two models reach about the same level of debt yet the Default-Reference model converges to that level faster and requires the government to pay higher yields, always, not only during the transition. In contrast, in NK-Default, the government internalizes that large capital inflows are distortionary, increasing the labor wedge and depressing domestic activity, and therefore endogenously decides to reduce borrowing and slow the time path of debt accumulation. Lenders price this reduction in borrowing, default risk, and associated debt dilution so that the NK-Default government faces more favorable terms of borrowing both state-by-state and in equilibrium, compared to the Default-Reference model. Spreads are lower by about 50 basis points on average in NK-Default, based on the simulation results reported in Section 4.8.

Figure 5 plots spreads in the two models, NK-Default (solid blue) and Default-Reference (dashed red), both in terms of schedules, for arbitrary borrowing B' in panel (a), and in equilibrium, as a function of debt B, in panel (b). Borrowing is everywhere more expensive in the Default-Reference model, mainly driven by its more aggressive borrowing, leading to more default risk and more debt dilution.



(a) Spread Schedule (b) Equilibrium Spreads

Figure 5: Spread Curves

4.7 Impulse Response Functions

In Section 4.4 we described the model's behavior in terms of the level of debt. Here, we turn to the response to shocks, with the use of impulse response functions. We consider i) a contractionary monetary shock m and ii) a low productivity realization z. To highlight the mechanism of default risk and time-consistency problem of the government, we contrast our model with the NK-Reference model introduced in Section 4.5, a version of Gali and Monacelli (2005) with incomplete markets.

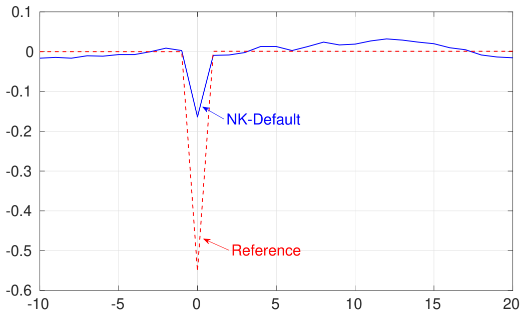
We construct the impulse response functions in our nonlinear model following Koop et al. (1996). We simulate a panel of $50,000$ time series for 5000 periods. For the first 4950 periods, the shocks follow their underlying Markov chains so that the cross-sectional distribution converges to the ergodic distribution of the model. In period 4951, the *impact period*, normalized to 0 in the plots, we alter shocks for all units by the same amount. From period 4952 onward, the shocks resume following their Markov processes. The impulse responses plot the average, across the time series.[6]

Monetary Shocks. Figure 6 plots the responses to a monetary shock m of aggregate output, domestic consumption, imports, inflation, the nominal interest rate, terms of trade, debt, and spreads.[7] The solid blue lines are for the benchmark NK-Default model while the red dashed lines are the NK-Reference model.

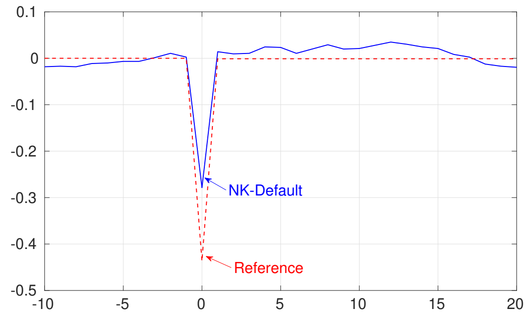
The monetary shock leads to an increase in the nominal interest rate of roughly 1.5% on impact, as shown in Panel (e). Due to this tightening of monetary policy, on impact, aggregate output in Panel (a) declines by 0.2%, domestic consumption drops by 0.25%, and imports shrinks almost 1.0%. The increased nominal interest rate leads to a reduction in inflation of about 0.3%. Terms of trade depreciate by about 0.1% and the spread declines 0.1% due to lower government borrowing, in panel (g).

[6]The impulse responses are computed over all 50,000 series, including those with defaults. Discarding defaults from the cross-section average does not alter the properties of the IRFs.

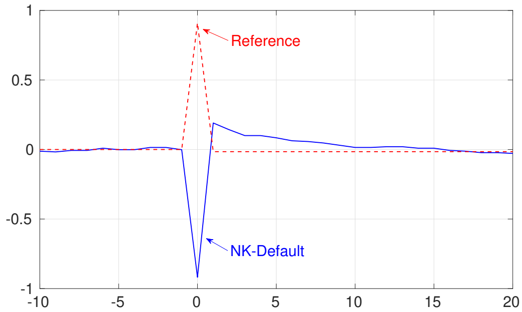
[7]Recall that the monetary shock we consider are one-time zero-probability innovations. IRFs for non-degenerate iid Normal monetary shocks look similar.



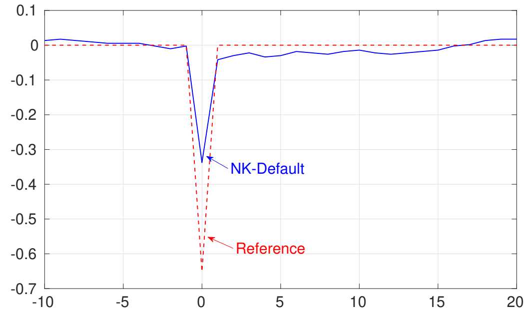
(a) Output



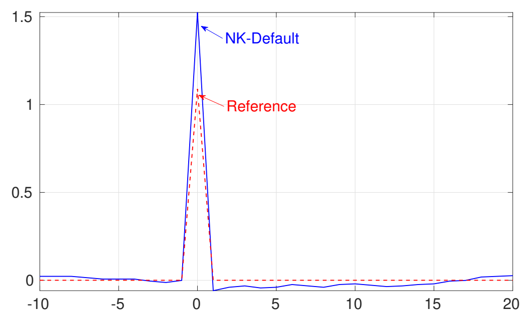
(b) Domestic Consumption (C)



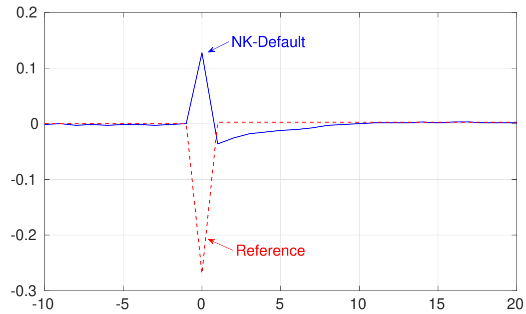
(c) Imported Consumption (C^f)



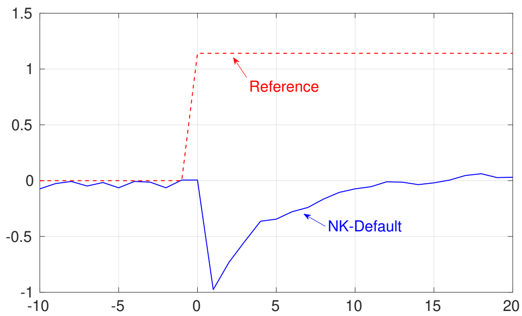
(d) Inflation (π)



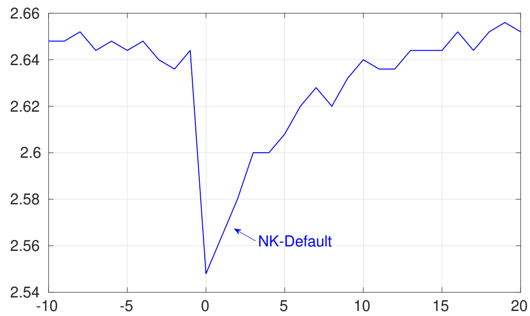
(e) Nominal Interest Rate (i)



(f) Terms of Trade (e)



(g) Debt (B)



(h) Spread

Figure 6: Impulse Responses to Monetary Shock

These responses of output, domestic consumption, inflation, and of the nominal rate are qualitatively similar to those in the standard New Keynesian small open economy models, as seen from the comparison with the dashed lines of the Reference model in Figure 6. In both models, tight monetary policy depresses consumption. The reduction in domestic consumption lowers the unit cost of production and leads to low inflation and output, in this demand-driven economy.

The two models differ in the behavior of imports, terms of trade, borrowing, and sovereign spread. The Reference model provides almost perfect risk sharing and, following a contractionary monetary shock, the country increases its debt dramatically. As shown in Panel (c), imports in the reference model actually increase. Higher inflows of foreign goods, supported by borrowing, lower the relative price of foreign goods and lead to an appreciation of the terms of trade.

In our benchmark model, borrowing also responds to the incentive to smooth imported consumption but the presence of sovereign debt with default risk restricts international borrowing possibilities and enables the government to alter the private equilibrium. In panels (h-g), we show that government's borrowing and associated spreads fall on impact. Increasing nominal interest rates worsens monetary distortions and, as seen in the Euler equation in Section 3.2, such worsening disciplines borrowing. The government responds by reducing debt and spreads.

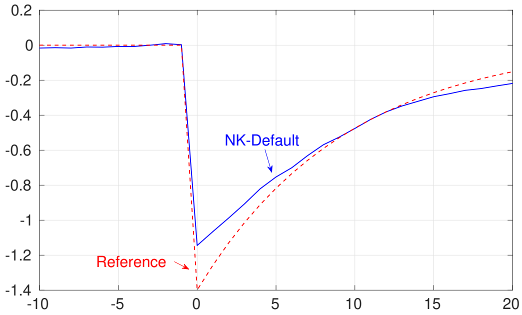
The terms of trade present different dynamics across the two models. In the Reference, the standard Unconvered Interest Rate Parity (UIP) logic calls for an appreciation on impact, followed by expected depreciation. In NK-Default, the monetary and default distortions outweigh this logic and the government adjusts borrowing enough so that terms of trade depreciate on impact, followed by expected appreciation.

The decline of inflation is larger on impact and shorter-lived in the Reference model relative to the benchmark. These different dynamics imply a higher resource cost from inflation in the Reference. The additional smoothing of inflation in the benchmark is enabled by the reduction in borrowing: carrying less debt into next period implies lower debt service, less real activity aimed at exporting and thus lower inflation.

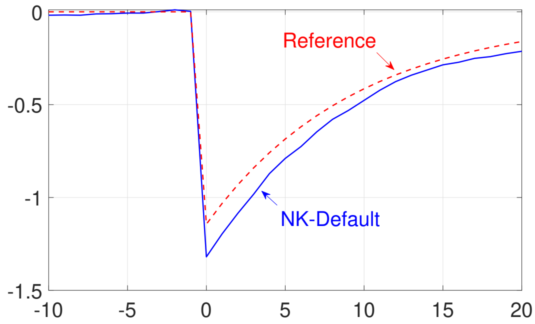
After the impact period, the allocations and prices recover in both models. Nevertheless, monetary shocks lead to persistent dynamics in our model, because of the endogenous dynamics of debt, due to default risk and incentives to smooth inflation and output.

Productivity Shocks. Figure 7 plots the impulse responses for the same variables to a productivity shock z, a decline of about 1.0%, for both NK-Default and the Reference model. We first discuss the responses in the Reference model and then turn to the behavior of our model.

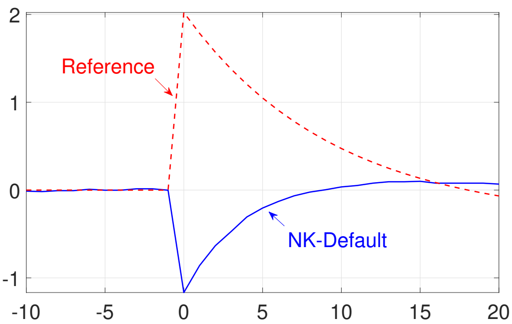
The red dashed lines in Figure 7 plots the IRFs of the reference model. Panels (a), (b), and (c) plot aggregate output, domestic consumption, and imports respectively. As is standard, low productivity reduces output and consumption. In Panel (d) we see that, as is typical in



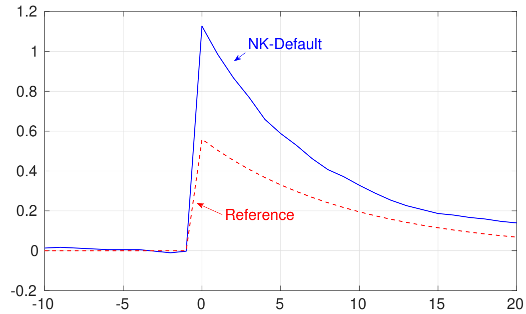
(a) Output



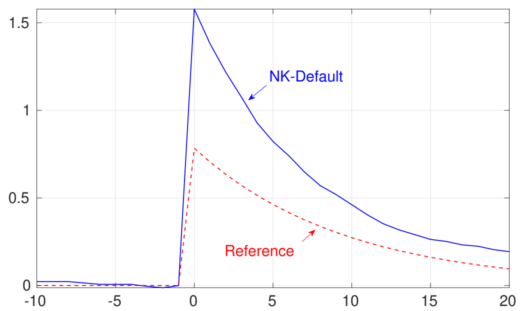
(b) Domestic Consumption (C)



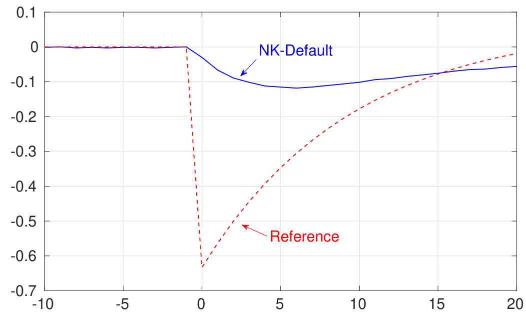
(c) Imported Consumption (C^f)



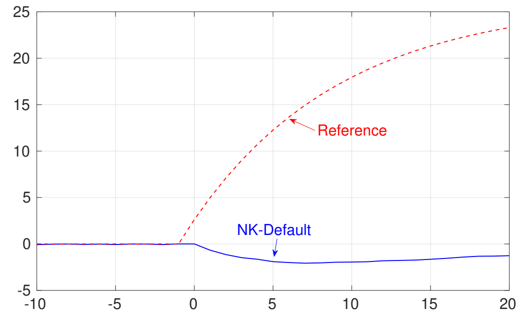
(d) Inflation (π)



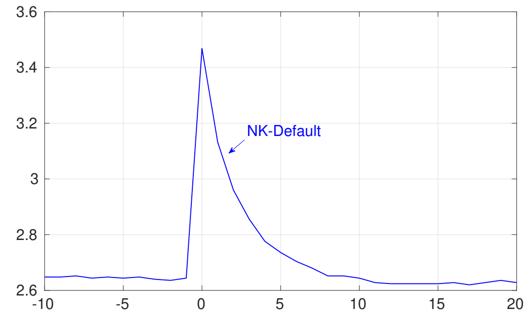
(e) Nominal Interest Rate (i)



(f) Terms of Trade (e)



(g) Debt (B)



(h) Spread

Figure 7: Impulse Responses to Productivity Shock

New Keynesian models, inflation rises with low productivity, about 0.5%, since firms face higher unit costs. The response of the domestic nominal rate is in panel (e). It rises about 0.8% due to the response called for by the interest rate rule, when facing higher inflation. Given the loose borrowing schedule, unaffected by the negative shock, the economy take on additional foreign debt (see panel (g)), to support imports. The inflow of foreign goods leads to an appreciation of the terms of trade, i.e. e decreases.

Qualitatively, the benchmark model has similar responses in output, consumption, inflation and nominal interest rate. The magnitudes, however, are different. In the benchmark, GDP falls less, 1.1% versus 1.4% for the reference. Imports falls by 1.0%. Inflation and the nominal rate increase more on impact, roughly an extra 1.0% for both. Moreover, the terms of trade appreciation is much more modest in the benchmark, in contrast to the Reference model. These dynamics reflect the presence of the default risk in our model. Low productivity tightens the bond price schedule and leads to higher spreads even as the government borrows less, in the solid blue lines of panels (g) and (h). The need to reduce indebtedness, contrasted with the expansion in debt in the Reference, will imply higher use of the labor input and thus a smaller drop in output. Such relative high labor usage in the benchmark drives up the unit cost, resulting in higher inflation. In response to the increased inflation in the benchmark, the monetary authority raises the nominal rate even higher than in the Reference model.

The dynamics of the terms of trade in Panel (e) are also shaped by tight external financial conditions, due to default risk. With high spreads, the government cuts its debt and country export more. An alternative way to view the pattern of appreciation is that with fewer foreign goods flowing into the country domestic goods are relatively abundant.

In summary, low productivity leads to a decline in output and consumption, both domestic and imported. Productivity shocks generate positive co-movements between inflation, the nominal interest rates and bond yields, as sovereign spreads are countercyclical. Moreover, the comparison of the two models highlights the transmission of sovereign spread to monetary policy. Sovereign default risk induces additional volatility in consumption and inflation, so that the interest rate rule calls for more aggressive monetary tightening, i.e. larger responses of the nominal rate to shocks.

4.8 Second Moments

Table 4 reports the mean and standard deviation of key variables as well as their correlations with the sovereign spread and output in the data, our benchmark model NK-Default, and two reference models: NK-Reference, a version of our model without sovereign borrowing but with pricing frictions and monetary policy, and a Real-Default version, with the same borrowing and default structure as NK-Default but without monetary frictions. We calibrate NK-Default to the data, as detailed in Section 4.2, and use the same parameter values in the other two models. Statistics are computed over simulations of length 50,000 periods, excluding periods in default or if the country was in default at any time in the previous 5 years. This selection criterion ensures that statistics are not biased by dynamics following the return to market access with zero debt, after default. We confirm that results are not sensitive to this selection criterion.

Mean	Data (%)	NK-Default	NK-Reference	Default-Reference
Inflation	5.9	5.9	6.1	0.0
Domestic Rate	11.2	11.1	11.5	5.3
Spread	2.6	2.6	—	3.2
Standard Deviation				
Inflation	1.8	1.8	1.0	0.6
Domestic Rate	2.2	2.5	1.3	1.8
Spread	0.9	0.9	—	0.8
Output	1.9	1.9	2.8	2.1
Consumption Aggregate	1.8	2.0	2.0	2.2
Trade Balance	0.9	0.3	15.8	0.5
Nominal Exchange Rate	14.7	2.4	2.0	1.9
Correlation with Spread				
Inflation	59	60	—	−1
Domestic Rate	59	64	—	18
Output	−62	−60	—	−42
Trade Balance	61	35	—	33
Nominal Exchange Rate	51	45	—	−1
Correlation with Output				
Inflation	−16	−88	−39	7
Domestic Rate	−23	−96	−48	−60
Trade Balance	−77	−18	90	−23
Nominal Exchange Rate	−18	−62	−13	7

Table 4: Moments: Data, NK-Default, and Reference Models

Overall, our moment matches well the Brazilian data. The mean inflation, nominal interest rate and spread are close to the data. The volatility of inflation and the nominal rate are comparable but the latter is slightly higher in the model. Throughout "inflation" refers to CPI inflation, which is the series available in Brazilian data. The volatility of the spreads in the model is in line with the EMBI data. Both the model and the data exhibit inflation and nominal rates that are positively correlated with spread, correlations of comparable magnitude, even though these moments were not targeted in the calibration. As is usual in the sovereign default literature, spreads and the trade balance are countercyclical.

Table 4 also reports the moments from the NK-Reference model. Here without default risk, spreads are trivially zero. By construction, the Reference model has the same average inflation and nominal rate as the benchmark. The volatilities of the nominal interest rate and inflation are, however, only about half of those in the NK-Default. This comparison shows that in an environment with default risk, a central bank targeting inflation must need to implement a more aggressive interest rate policy. This is because default risk impacts the monetary distortion, reflected by the labor wedge, as discussed in Section 4.5 and inflation responds more to the productivity shock. The end result is that a larger response by the nominal interest rate is needed to keep inflation close to target, as illustrated by the impulse responses in Section 4.7.

The comparison with the Default-Reference model confirms that without the disciplining effect of the monetary friction, the government borrows more aggressively and faces higher spreads. Note that spreads are, on average 50 basis points higher in the Default-Reference model compared to NK-Default and the Trade Balance to Output ratio is almost twice as volatile. In the Default-Reference model we can use the equilibrium allocation to back out an implied domestic interest rate: we find that it is positively correlated with spreads, as in NK-Default, but this correlation is much weaker (0.18 versus 0.64, compared to 0.59 in the data, an value not targeted in the calibration). Similarly, in the Default-Reference model we can define CPI inflation, due to movements in the terms of trade, even absent PPI inflation, but here CPI inflation is uncorrelated with spreads, while NK-Default replicates this strong correlation in the data.

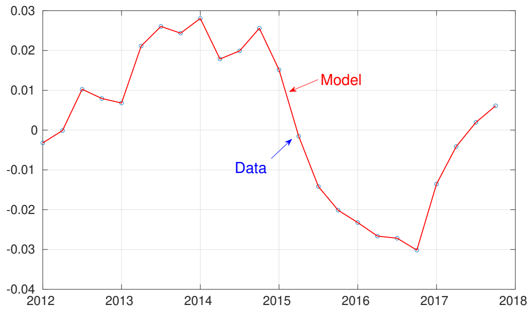
4.9 Brazil Event

We use our framework as a "laboratory" to understand the Brazil 2015 recession and to simulate counterfactual policies, in order to better showcase the interactions between spreads, inflation, and nominal rates. The model produces similar increases of nominal rate, inflation, and spreads as in the data. To highlight the impact of monetary policy on sovereign default risk, we conduct a counterfactual experiment where the nominal rate is fixed at its long-run $\bar{i}$ level at the start of the recession. The experiment shows that if the central bank would have been more dovish during this episode, Brazil would have faced not only higher inflation but also much higher spreads.

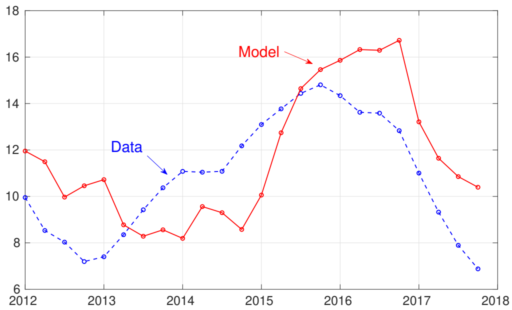
To replicate the Brazilian event in the model, we design a path of productivity shocks such that the time path of output in the model replicates the one in the data. We fix the initial level of debt to the mean of the limiting distribution and we hold the monetary shock m fixed at a neutral value of 1. We then compare the predictions of the model for inflation, the nominal interest rate, government spreads, and nominal exchange rates to the data.

The blue dashed lines with circle markers in Figure 8 represent the series in the data. Brazil experienced a recession from 2014 to late 2016, with GDP contracting from 3% above trend to 3% below trending, a 6% decline in total. It then recovers starting in 2016Q3. During this period, inflation increases by 4%, the nominal rate increases by 2%, and spreads rise from about 2% to 5%. When GDP recovers after 2017, inflation, nominal rate, and spread all fall.

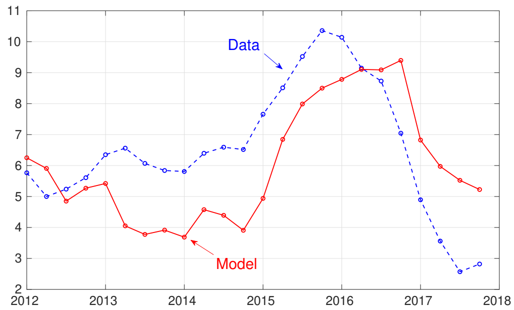
The solid red lines in Figure 8 are the corresponding series in the model. To match the



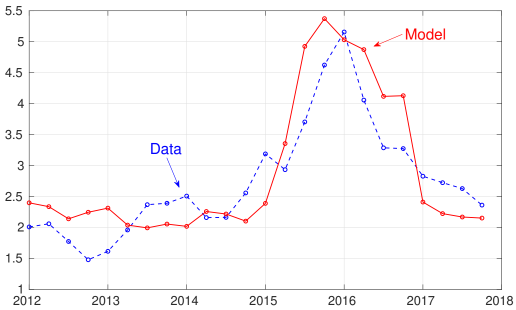
(a) Output



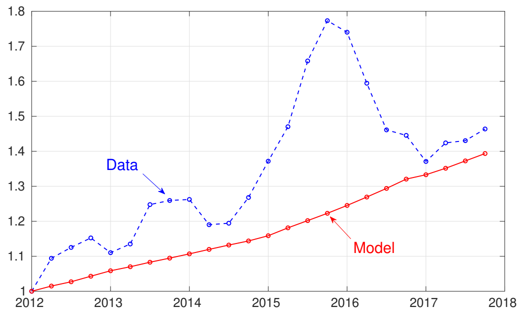
(b) Nominal Interest Rate



(c) Inflation



(d) Sovereign Spread



(e) Nominal Exchange Rate

Figure 8: Event: Brazil

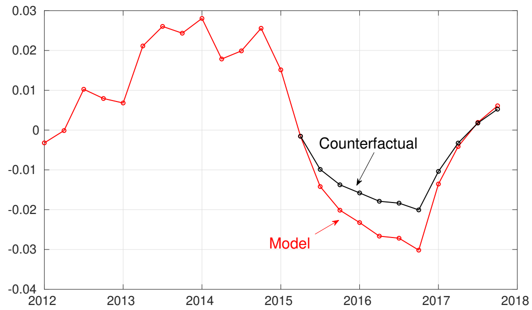
dynamics of output, the model requires that the underlying productivity shock first decreases from 2014 to late 2016 and then recovers. This implies that during the recession the unit cost of production increases, leading to an increase in inflation. Monetary policy responds to this high inflation with a hikes in the nominal rate. Low productivity also drives up the sovereign's default risk. Quantitatively, the model matches the rise of inflation and spreads during the recession, around 4% for inflation and 3% for the spread. The model also produces the observed recovery from 2016Q3 onward, inflation decreases about 4% and spread drops by about 3%.

The model delivers an overall, cumulative nominal depreciation of the exchange rate comparable with the data, of about 50%, but misses the high volatility of the data. Our model's Armington trade structure exhibits the real exchange rate puzzles widely discussed in the literature, e.g. Obstfeld and Rogoff (2001).

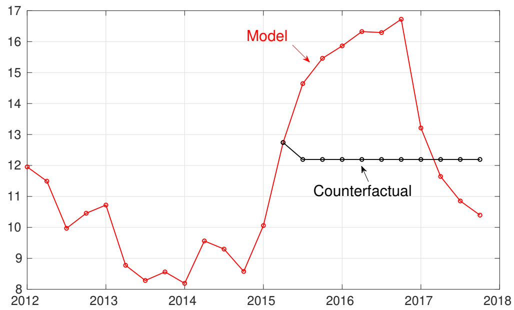
Overall the model captures well the broad patterns in the data: the joint dynamics of spreads and inflation, the movement with the domestic nominal rate, and the rising default risk in recessions. In the model this behavior is driven by low productivity, driving unit costs up and increasing default temptations for the fiscal authority.

To understand the impact of pricing frictions and monetary policy on fiscal policy and the resulting sovereign default risk, we conduct a counterfactual experiment with a dovish central bank. In this alternative scenario, instead of following the policy called for by its interest rate rule and the inflation target, the central bank keeps a low nominal interest rate, similar to its 2015, neutral level following the start of the recession. We feed in a sequence of monetary shocks m so keep nominal rates near this initial value. The counterfactual series are plotted in black in Figure 9 while the original NK-Default series are again in solid red. These expansionary monetary shocks induce lower nominal interest rates and thus stimulate consumption. This higher demand increases the unit cost of production, which in turn generates high inflation. In late 2016, the inflation rate in the counterfactual scenario would be about 2 percent higher than the benchmark case. Lower nominal interest rate from the dovish central bank also leads to a higher spread, since the government increases its borrowing, to shift future consumption to the current period. The exchange rate therefore depreciated even further. This counterfactual highlights the disciplining role of monetary frictions for sovereign borrowing and default risk.

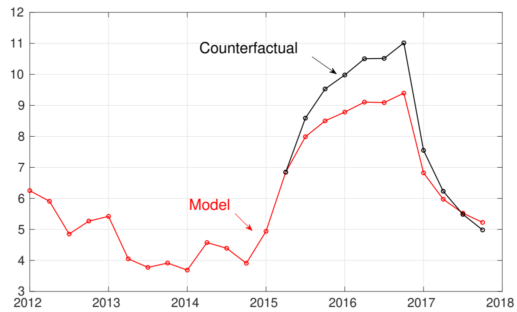
In summary, our model matches well the Brazilian downturn around 2016 and 2017 in both real and nominal terms. The counterfactual highlights the role of monetary frictions in limiting borrowing and moderating crisis events. Our model provides a good laboratory to understand the interplay between monetary and fiscal policy in the presence of default risk. Had Brazil's central bank deviated from its pursuit of price stability, the recession would have been milder but at a cost of much higher inflation and a deeper debt crisis.



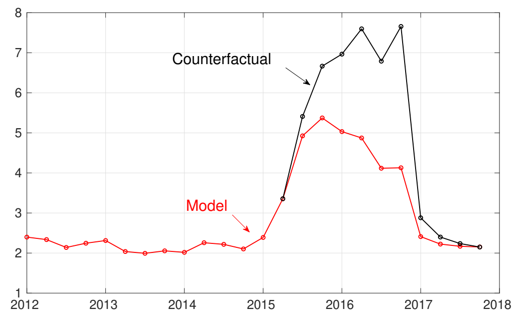
(a) Output



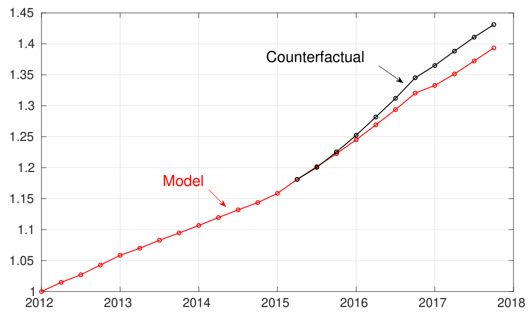
(b) Nominal Interest Rate



(c) Inflation



(d) Sovereign Spread



(e) Nominal Exchange Rate

Figure 9: Event: Counterfactual Experiment

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A Numerical Implementation

A.1 Computation with Taste Shocks

We compute the model using discrete choice methods, following Dvorkin et al. (2018) and Gordon (2018), who adapt tools frequently used in structural applied work for the study of sovereign default with long-term debt. Chatterjee and Eyigungor (2012) follow a related strategy, also perturbing the borrowing choice (B'), in order to address the convergence problems inherent in models with long-term debt.

We restrict the choice of B' to be in a discrete set and associate each option with an iid taste shock distributed Gumbel (Extreme Value Type I). The government's problem becomes

$$\mathcal{W}\left(s, B, \langle \epsilon_{B'} \rangle\right) = \max_{B'} \left\{ \mathcal{J}(s, B, B') + \varrho_B \, \epsilon_{B'} \right\} \tag{40}$$

with

$$\mathcal{J}(s, B, B') \equiv u \left[C(s, B, B'), C^f(s, B, B'), N(s, B, B') \right] + \beta_g \mathbb{E}_{s'|s} V(s', B') \tag{41}$$

and where ϱ_B is a constant governing the relative importance of the taste shocks for the choice of B' and $\langle \epsilon_{B'} \rangle$ is a vector of taste shocks, one for each possible value of B' on the grid. As $\varrho_B \to 0$ we recover the unperturbed initial problem, with poor numerical convergence properties, while as $\varrho_B \to +\infty$ the taste shocks dominate and the choice of B' become uniform iid. Ex-ante, before taste shocks are realized, the choice probabilities are given by

$$\Pr\left(B' = x | s, B\right) = \frac{\exp\left[\mathcal{J}(s, B, x)/\varrho_B\right]}{\sum_{\tilde{x}} \exp\left[\mathcal{J}(s, B, \tilde{x})/\varrho_B\right]} = \frac{\exp\left[\left(\mathcal{J}(s, B, x) - \overline{\mathcal{J}}(s, B)\right)/\varrho_B\right]}{\sum_{\tilde{x}} \exp\left[\left(\mathcal{J}(s, B, \tilde{x}) - \overline{\mathcal{J}}(s, B)\right)/\varrho_B\right]} \tag{42}$$

with $\overline{\mathcal{J}}(s, B) = \max_{B'} \mathcal{J}(s, B, B')$ and government's value is

$$W(s, B) = \mathbb{E}_{\langle \epsilon_{B'} \rangle} \left\{ \mathcal{W}\left(s, B, \langle \epsilon_{B'} \rangle\right) \right\} = \overline{\mathcal{J}}(s, B) + \varrho_B \log \left\{ \sum_{B'} \frac{\mathcal{J}(s, B, B') - \overline{\mathcal{J}}(s, B)}{\varrho_B} \right\}. \tag{43}$$

$\overline{\mathcal{J}}(s, B)$ is the value the government would achieve if all the taste shock would be zero (or if the problem were unperturbed) while $W(s, B)$ is the expected value before the realization of the taste shocks. Panel (a) of Figure 10 plots an example of choice probabilities, associated with state $\langle z = 1, B = 0.25 \rangle$. The probability mass is tightly centered around the B' that maximizes $\mathcal{J}(s, B, B')$.

Additionally, we perturb the default decision in a similar fashion. At the start of each period, the government observes default decision taste shocks and decides accordingly:

$$V(s, B) = \mathbb{E}_{\epsilon_{\text{Repay}}, \epsilon_{\text{Default}}} \max \left\{ W(s, B) + \varrho_D \, \epsilon_{\text{Repay}}, \; W_d(s) + \varrho_D \, \epsilon_{\text{Default}} \right\} \tag{44}$$

As a consequence, if state $\langle s, B \rangle$ is realized, the government chooses default with probability

$$\Pr(D = 1|s, B) = \frac{\exp\left[W_d(s)/\varrho_D\right]}{\exp\left[W_d(s)/\varrho_D\right] + \exp\left[W(s, B)/\varrho_D\right]} \tag{45}$$

For values of ϱ_D greater than zero, the default probability is everywhere nondegenerate, although often numerically indistinguishable from zero or one. This induces bond price schedules that are smooth in the borrowing choice B', further aiding numerical convergence. Panel (b) of Figure 10 plots "borrowing Laffer curves" ($q(s, B')B'$) for various levels of the productivity shock.

In the model augmented with taste shocks, the expression for the bond price schedule becomes

$$q(s, B') = \frac{1}{1 + r^*} \mathbb{E}_{s'|s} \Pr(D = 0|s', B') \left\{ r^* + \delta + (1 - \delta) \sum_{B''} \Pr(B''|s', B') q(s', B'') \right\} \tag{46}$$

The expectation functions $M(s, B')$ and $F(s, B')$ are updated analogously.

A.2 Algorithm

The model is subject to an AR(1) productivity shock z, which we discretize over a grid with $\#z = 21$ points spanning ± 3 standard deviations of the unconditional distribution. We also allow for a zero-probability shock to the interest rate rule m, which we discretize over $\#m = 7$ points spanning $\pm 1.5\%$. The m shock is iid, with $\Pr(m = 1) = 1$ and $\Pr(m \neq 1) = 0$. We use this "MIT-style" shock to study the consequences of unexpected monetary tightening in the quantitative analysis. The B grid consists of $\#B = 250$ points equally spaced over $[0, 0.5]$.

The algorithm proceeds as follows

1. We start with initial guesses for the value functions V_0, W_0^d and the bond price schedule q_0, together with guesses for the F_0 and M_0 functions and the default and borrowing policies. We assume the probability of default is 1 and $B' = B$ with probability one, everywhere in the state space.

2. We solve for for the Private and Monetary Equilibrium everywhere in the state space, for arbitrary B'. We restrict attention to B' values that do not induce "too large" capital inflows or outflows, for which a PME might not exist and confirm that this restriction does not bind in equilibrium: $| - (r^* + \delta)B + q(s, B')(B' - \delta B)| < 0.1$.

   We solve the PME via root-finding using Powell's hybrid method, on a system of two equations in two unknowns, C^f and N:

   (a) Use current guess of $\langle C^f, N \rangle$ and the capital inflow $-\lambda B + q(s, B')(B' - \delta B)$, we compute the terms of trade e from the Balance of Payments condition.

   (b) We compute the implied level of exports X associated with the Terms of Trade e.

(c) Given C^f and e we can recover domestic consumption C from the relative consumption condition.

(d) Given C and the government's borrowing choice B' we compute the domestic nominal rate i from the domestic Euler equation.

(e) Given i we use the interest rate rule to compute the level of PPI inflation π.

(f) We use these quantities to compute equation residuals for the New Keynesian Philips Curve and the domestic resource constraint.

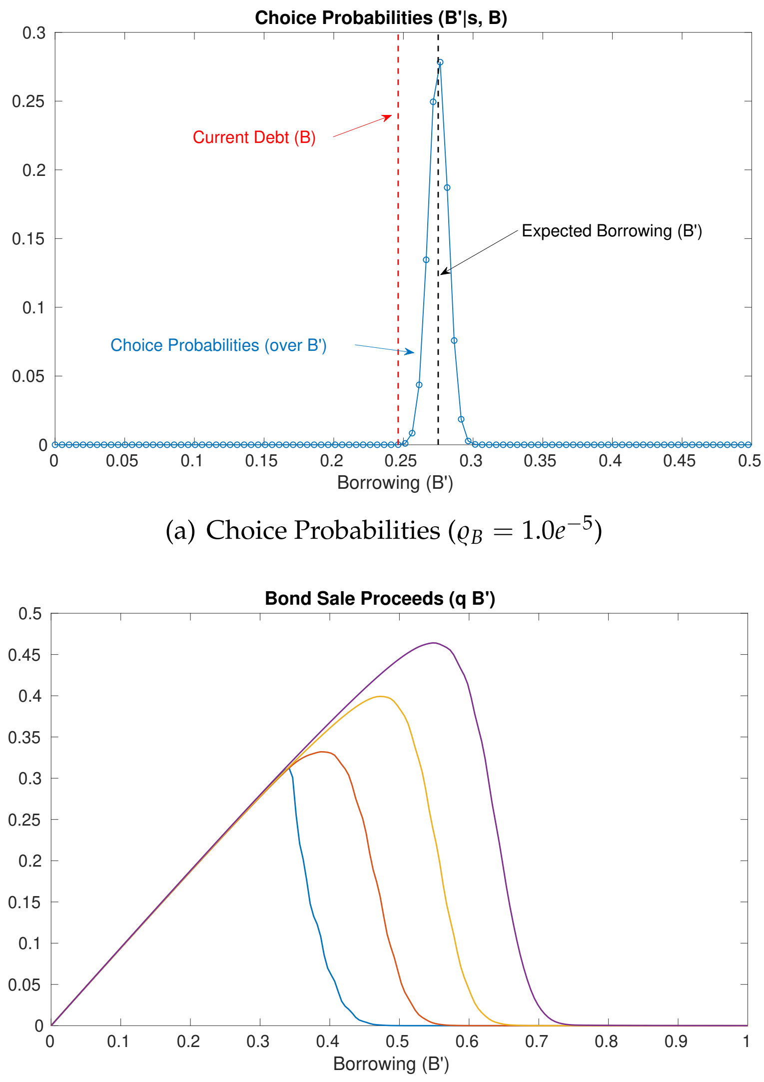
The solution to the PME yields policy functions $C(s, B, B')$, $C^f(s, B, B')$, $N(s, B, B')$, $\pi(s, B, B')$, $i(s, B, B')$, $e(s, B, B')$.

3. We solve the PME in default similarly. In particular, in default trade is balanced and the capital inflow term is zero, and productivity is penalized $h(z) < z$. The solution constitutes policy functions in default: $C_d(s)$, $C_d^f(s)$, $N_d(s)$, $\pi_d(s)$, $i_d(s)$, $e_d(s)$.

4. Using PME results, we compute the value of the government in each state (V) and in default (W_d) and derive choice probabilities for the B' policy and default probabilities.

5. Given borrowing and default policies (probabilities) we update the bond price schedule q and the expectation functions M and F.

6. We check for the convergence of the bond price schedule, value functions, and expectation functions. We stop if values are closer than $1e^{-7}$ and prices closer than $1e^{-5}$ in the sup norm, otherwise we fully update and iterate.



(a) Choice Probabilities ($\varrho_B = 1.0e^{-5}$)

(b) Bond Price Schedules ($\varrho_D = 1.0e^{-4}$)

Figure 10: Computation with Taste Shocks