Deadly Debt Crises: COVID-19 in Emerging Markets*

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Current version April 2021. First version May 2020.

Abstract

The coronavirus pandemic has severely impacted emerging markets by generating a large death toll, deep recessions, and a wave of sovereign defaults. We study this compound health, economic, and debt crisis and its mitigation by integrating epidemiological dynamics into a sovereign default model. The epidemic leads to an urgent need for social distancing measures, a large drop in economic activity, and a protracted debt crisis. The presence of default risk restricts fiscal space and presents emerging markets with a trade-off between mitigation of the pandemic and fiscal distress. A quantitative analysis of our model accounts well for the dynamics of deaths, social distance measures, and sovereign spreads in Latin America. In the model, the welfare cost of the pandemic is higher because of financial market frictions: about a third of the cost comes from default risk, compared with a version of the model with perfect financial markets. We study debt relief programs through counterfactuals and find a compelling case for their implementation, as they deliver large social gains.

Keywords: default risk, sovereign debt, pandemic mitigation, COVID-19, debt relief, official lending
JEL classification: F34, F41, I18

*We thank Mark Aguiar and Manuel Amador for their comment, and Hayagreev Ramesh and Simeng Zeng for excellent research assistance. We also thank Stony Brook Research Computing and Cyberinfrastructure and the Institute for Advanced Computational Science at Stony Brook University for access to the high-performance SeaWulf computing system, which was made possible by a $1.4M National Science Foundation grant (#1531492). The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. *Contact information:* arellano.cristina@gmail.com; yanbai06@gmail.com; mihalache@gmail.com

1 Introduction

The coronavirus pandemic has brought enormous challenges for world economies. To control this highly contagious and deadly disease, countries have relied on mitigation measures that limit social interactions, while experiencing severe contractions in economic activity. Many governments have also engaged in large fiscal transfers to support private consumption in response to the recession. In emerging markets such transfer programs have been much smaller because their governments have limited fiscal space, owing to their chronic problems with public debt crises.[1] In fact, during the pandemic, many countries have defaulted on their sovereign debt (among them Argentina, Ecuador, Ethiopia, and Lebanon), and all emerging markets have experienced increased sovereign spreads. This paper studies the interactions between public debt and the epidemic and shows that susceptibility to debt crises magnifies the economic and health costs of the pandemic.

We develop a framework that integrates standard epidemiological dynamics into a model of sovereign debt and default. The epidemic triggers a health crisis, with time paths of infected and deceased individuals. The government in our model uses public debt but lacks commitment to repay, and thus it might choose to default, with varying intensity and duration. The economy responds to the epidemic with social distancing measures that save lives but depress output. The government borrows to support consumption during the epidemic, but sovereign default risk limits its ability to do so. The tepid expansion of government borrowing is nevertheless expensive for the economy because it increases the likelihood of a lengthy and costly debt crisis. Default risk increases the welfare cost of the epidemic, because by constraining consumption, it increases the cost of social distancing measures needed to fight the health crisis. We apply our framework to data from Latin America, a region that has experienced a severe COVID-19 outbreak. We fit the model to time series of Google Mobility data and COVID-19 daily deaths. We find that the welfare cost of the epidemic is large, about 28% of annual output for the country, and also about 7% for its lenders, which hold its outstanding debt upon the outbreak. These costs reflects an elevated death toll of 0.16% of the population, a prolonged debt crisis lasting four years, and significant output losses. We find that sovereign default risk accounts for about a third of these costs.

The epidemiological model is the standard susceptible-infected-recovered (SIR) framework. The epidemic starts when an initial fraction of the population becomes infected. New infections result from the interactions of those currently infected with individuals who are susceptible to the disease, as well as from the degree to which the virus is contagious. The infected individuals transition eventually to either a recovered state or a deceased state. We follow Alvarez, Argente, and Lippi (2020) and assume that the death rate depends on the fraction of the population that is currently infected and that social distancing measures limit the growth of new infections. The sovereign debt and default framework we adopt follows the one in Arellano, Mateos-Planas, and Ríos-Rull (2019). The sovereign of a small open economy borrows internationally and chooses the intensity of default every period, endogenously determining the duration of the default episode. A fraction of the defaulted debt accumulates; it is capitalized in the stock of outstanding debt, while new credit is endogenously restricted. Partial defaults

1. Gourinchas and Hsieh (2020) were among the first to issue a warning about precarious debt conditions in emerging markets and their potential impact on fighting the pandemic. They argue in favor of vigorous international support and a debt moratorium.

in this framework amplify shocks and lead to persistent adverse effects on the economy. We consider a centralized problem with a sovereign that values the lives and consumption of the population. The sovereign decides on borrowing, partial default intensity, and social distancing measures (also referred to as "lockdowns") to support consumption and manage the infection dynamics with a goal of preventing deaths. In our framework, default risk responds to the epidemic and shapes its management.

We use a simplified version of our setup, with a finite horizon, to characterize more sharply the interactions between default risk and epidemic outcomes. Social distancing measures are an investment in lives and, as such, respond to consumption costs and domestic interest rates, which reflect the shadow cost of borrowing arising from default risk. We show that with perfect financial markets, the marginal cost of social distancing measures tends to be lower, because consumption is determined by the economy's lifetime income. With default, in contrast, lockdowns tend to be inefficiently loose because of the higher marginal cost of consumption arising from a lower lifetime income due to default costs and high domestic rates. We show that default risk leads to under-investment in lives and makes the epidemic more deadly.[2] We also show that the epidemic leads to an increase in default risk, because of additional incentives to borrow.

We evaluate the interaction between financial market frictions and epidemic outcomes in our quantitative model by comparing our baseline with results in two reference setups: perfect financial markets and financial autarky. The epidemic results in sizable loss of life in all economies, but with perfect financial markets, the economy can implement more stringent social distancing measures that can reduce the death toll to less than one-third of that in the baseline. Under financial autarky, the death toll is about 20% higher than in the baseline. The markedly different health outcomes, together with the degree to which financial markets can support smooth consumption during the episode across these models, result in sizable differences in the welfare costs of the epidemic.

The fact that financial conditions greatly impact outcomes during the epidemic suggests that international assistance programs can deliver considerable benefits to emerging markets burdened by default risk during the COVID-19 pandemic. The International Monetary Fund, the World Bank, the Inter-American Development Bank, and other international organizations are sponsoring debt relief programs to help countries fight the pandemic. We use our model to conduct two counterfactual experiments to evaluate such debt relief initiatives. The first program we consider is a default-free, long-term loan by a financial assistance entity. We find that it has large social benefits, increasing the welfare in the baseline by 7.5% for the country and 4.7% for its lenders. These gains arise from a reduction in deaths and a much milder debt crisis, which are due to more efficient mitigation measures and less reliance on defaultable debt. The second program consists of a voluntary restructuring between the country and its private creditors. We find that upon the outbreak of the epidemic, at our baseline parameterization, the economy and its lenders will voluntarily agree to reduce the debt level by close to 10% of output, without affecting the value of debt to lenders. The increase in the market price of the outstanding debt compensates the loss from holding fewer units. In turn, the economy naturally gains from such a voluntary reduction in its level of debt, by about 11% of output.

Finally, our work makes a methodological contribution. We develop a framework that integrates the

2. Our finding that default risk discourages social distancing measures relates to the debt overhang literature, which has argued that indebtedness can depress investment, as in Aguiar and Amador (2011).

dynamics of defaultable debt with those of epidemiological status in the population. We set up and solve a Markov problem, in which the government's choices over debt and social distancing measures affect the endogenous evolution of four state variables—namely, the debt and three population groups: susceptible, infected, and recovered. The sovereign lacks commitment and makes current choices, taking as given all future policies. We provide an algorithm that can be adapted to other applications of epidemiological dynamics in settings with time-varying endogenous aggregate state variables.

The remainder of this section provides a brief review of the relevant literature. The rest of the paper proceeds as follows. Section 2 lays out the structure of our model. Section 3 focuses on a simpler version of our setup, which enables us to highlight key interactions between deaths, mitigation of the epidemic, and default. Section 4 reports the results for the quantitative analysis of our model, including the data used to discipline it, counterfactual experiments across alternative financial market arrangements, and the evaluation of debt relief programs. Section 5 concludes and sets directions for future work.

Literature. Our paper contributes to the fast-growing literature that studies the COVID-19 epidemic and its interactions with economics. Atkeson (2020) was the first to introduce economists to the classic SIR epidemiology model and use it to study the human cost of the COVID-19 epidemic for the United States. Alvarez, Argente, and Lippi (2020) and Eichenbaum, Rebelo, and Trabandt (2020) study optimal mitigation policies in simple production economies, in which the epidemic dynamics follow a SIR model. Their results highlight the trade-off inherent in lockdowns: they save lives but are costly in terms of economic output. Our epidemiological model follows Alvarez, Argente, and Lippi (2020) setup, but adds consumption smoothing incentives as in Eichenbaum, Rebelo, and Trabandt (2020).

A growing literature considers the role of heterogeneity in the COVID epidemic. Glover, Heathcote, Krueger, and Ríos-Rull (2020) delve into crucial distributional considerations, as the old are more at risk from the epidemic, yet the young endure most of the economic costs from lockdowns. They find that social distancing and lockdowns are used more extensively by governments with better ability to redistribute. Acemoglu, Chernozhukov, Werning, and Whinston (2020) study lockdowns in environments with multiple ages and sectors. They find that smart mitigation strategies that target the old and at-risk population are most helpful. Baqaee, Farhi, Mina, and Stock (2020) and Azzimonti, Fogli, Perri, and Ponder (2020) study how the network structure of sectors and geography can be exploited in the design of optimal mitigation policies. Guerrieri, Lorenzoni, Straub, and Werning (2020) show that negative supply shocks in one sector such as COVID can depress aggregate demand in settings with multiple sectors and sticky prices. These papers focus on the epidemic's costs for advanced economies and have abstracted from the additional challenges in emerging markets. Our paper's emphasis is on the additional cost that the epidemic imposes on emerging markets in terms of the resulting debt crises, and to highlight our contribution we have abstracted from additional heterogeneity considerations. We view our work as complementary to these findings.

A few papers do share our focus on the impact of COVID-19 on emerging markets. Hevia and Neumeyer (2020) highlight the multifaceted nature of the pandemic, a tremendous external shock for emerging markets that includes collapsing export demand, tourism, remittances, and capital flows. Çakmaklı, Demiralp, Kalemli-Özcan, Yesiltas, and Yildirim (2020) focus on international input-output linkages as well as sectoral heterogeneity, by constructing a SIR-macro model calibrated to the Turkish input-output structure, while abstracting from default risk. Espino, Kozlowski, Martin, and Sanchez (2020) study optimal fiscal and monetary policies for emerging markets in a sovereign default model and model COVID-19 as an unexpected combination of shocks. Similar to our results, they find that default risk increases as a result of the epidemic. Different from us, they do not consider explicitly epidemiological dynamics and hence their framework is silent on the health crisis.

The dynamic debt and default framework at the core of our work builds on the earlier contributions by Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), Arellano (2008), and Chatterjee and Eyigungor (2012). We adopt the more recent approach in Arellano, Mateos-Planas, and Ríos-Rull (2019), who model debt crises with partial default and thus an endogenous length of the crisis.This framework gives meaningful dynamics during default episodes that replicate data, as defaulted debt accumulates over time and the length of the episode depends on the depth of the recession. Our quantitative evaluation of debt relief proposals contributes to the literature on debt buybacks. Like Bulow and Rogoff (1988) and Aguiar, Amador, Hopenhayn, and Werning (2019), we find that international lenders would benefit from debt buybacks during the COVID-19 epidemic through capital gains. Nonetheless, we emphasize that the gains to the country are large and positive because reducing debt overhang can considerably shorten and lessen the debt crisis and save on output costs from default. Furthermore, debt reduction allows the country to adopt stricter lockdown policies, which save lives. Our study of voluntary restructuring relates to the work of Hatchondo, Martinez, and Sosa-Padilla (2014), who evaluate similar proposals in a setup without epidemic dynamics. They also find scope for Pareto improvements, while focusing instead on the size of the shock.

2 Model

We consider a small open economy model with a continuum of identical agents and a government that borrows from the rest of the world, with an option to default on its debt. Output in the economy depends on labor input and productivity. We evaluate the dynamics of this economy after it is unexpectedly hit by an epidemic, COVID-19. The dynamics of infection and deaths follow a standard epidemiological SIR model. During the epidemic, a subset of the population endogenously transitions from being susceptible to being infected. The infected eventually either recover or die. The outcomes of the epidemic can be altered with social distancing measures, which we often refer to as “lockdowns” as shorthand.

We start by describing preferences, technology, the market for sovereign debt, and the default option. We then discuss the evolution of the disease and social distancing measures, and we formulate the dynamic problem during the epidemic. The outbreak starts when a subset of the population exogenously becomes infected.

2.1 Preferences and Technology

We consider preferences over consumption and lives. As in Alvarez, Argente, and Lippi (2020) and Farboodi, Jarosch, and Shimer (2020), the value increases with consumption per capita c_t and decreases with fatalities $\phi_{D,t}$. We assume each fatality imposes a loss of value χ. The lifetime value to the government is

$$v_0 = \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \chi \phi_{D,t} \right], \tag{1}$$

where β is the discount factor. The utility from consumption is concave and equals $u(c) = (c^{1-\sigma} - 1)/(1-\sigma)$, with σ controlling the intertemporal elasticity of substitution.

Output in the economy Y_t is produced using labor, which is impacted by social distancing measures. Agents are endowed with one unit of time, and hence total labor supply equals population N_t. During a lockdown of intensity L_t, each agent's labor input is reduced to $(1 - L_t)$.[3] The economy's output equals

$$Y_t = z_t \left[(1 - L_t) N_t \right]^{\alpha}, \tag{2}$$

where $0 \le \alpha \le 1$ and z_t is the economy-wide level of productivity, which depends on an underlying level $\tilde{z}$ and falls with government default.

2.2 Government Debt and Default

The government issues long-term debt internationally and lacks commitment over its repayment. We consider a sovereign default model, in which the government can choose to partially default on the debt every period and thus decides whether to start or end the default episode. We study long-term debt in a tractable way by considering random maturity bonds, as do Hatchondo and Martinez (2009). The bond is a perpetuity that specifies a price q_t and a quantity ℓ_t so that the government receives $q_t \ell_t$ units upon the sale, at time t. In each subsequent period, a fraction δ of the debt matures. Every period, conditional on not defaulting, each unit of debt calls for a payment of $\delta + r$.[4] The government can choose to default on a fraction d_t of the current payment owed, and it transfers to domestic households all of its proceeds from operating in international debt markets. The resource constraint of the economy is given by

$$N_t c_t + (\delta + r)(1 - d_t) B_t = Y_t + q_t \ell_t. \tag{3}$$

The equilibrium bond price q_t is determined by a schedule that depends on the debt level and epidemic demographics, because as we will see below, the likelihood of future default depends on these states.

In this model with accumulation of long-term defaulted debt, the debt due next period B_{t+1} depends not only on new issuance ℓ_t but also on the outstanding debt B_t and the share of debt on which the government defaults over time. Following Arellano, Mateos-Planas, and Ríos-Rull (2019), we assume that partial default reduces the current debt service payment to $(1 - d_t)(\delta + r)B_t$ but increases future debt obligation by a κ fraction of the defaulted payment. We annuitize these future debt obligations so that the next period's debt obligations increase by $\kappa d_t (\delta + r) B_t$. Default also depresses productivity to $z_t = \tilde{z}\gamma(d_t)$, where the function $\gamma(d_t)$ is decreasing and bounded between 0 and 1. The evolution of long-term debt is controlled by the new issuance ℓ_t, the outstanding debt that has yet to mature,

3. In this baseline model, we have assumed that all individuals, whether they are infected or not, can work equally well. It is easy to consider an extension in which infected individuals are subjected to a productivity penalty or completely unable to work.

4. We fix this payment level to normalize the risk-free bond price to 1. This normalization does not alter the maturity of the debt, only the units of our B_t variable.

$(1-\delta)B_t$, and any defaulted debt that is carried over:

$$B_{t+1} = \ell_t + [(1-\delta) + \kappa(\delta + r)d_t]\, B_t. \tag{4}$$

International lenders are risk neutral and competitive. They take as given the risk-free rate r, their opportunity cost. The bond price q_t compensates lenders in expectation for their losses due to future defaults,

$$q_t = \frac{1}{1+r}\left\{(\delta + r)(1 - d_{t+1}) + [1 - \delta + \kappa(\delta + r)d_{t+1}]\, q_{t+1}\right\}. \tag{5}$$

This expression reflects how partial default tomorrow d_{t+1} reduces the value that lenders get in period $t+1$ but increases the subsequent value to them as the defaulted payments accumulate at rate κ, to become due later.

2.3 Epidemic Dynamics

We now describe the outbreak of the epidemic and the subsequent dynamics, which build on the classic SIR structure of Kermack and McKendrick (1927). Following the outbreak of the disease, a subset of the population transitions endogenously from being susceptible to being infected and, eventually, to being either recovered or deceased. Thus, during the epidemic, the population N_t is partitioned in three epidemiological groups: susceptible, infected, and recovered. The mass of each group is denoted by μ_t^S, μ_t^I, and μ_t^R, respectively. We assume that the initial population size is 1. The total mass of the deceased is $\mu_t^D = 1 - N_t$. The epidemic starts when an initial mass of the population becomes infected exogenously, $\mu_0^I > 0$. The rest are susceptible, except possibly for a measure of agents already recovered $\mu_0^R \geq 0$, so that $\mu_0^S = 1 - \mu_0^I - \mu_0^R$.

The spread of the epidemic can be mitigated with lockdowns that limit social interactions, as in Atkeson (2020) and Alvarez, Argente, and Lippi (2020). These reduce labor input by L_t and social interactions by θL_t. The parameter θ controls the effectiveness of social distancing measures in preventing the spread of infection.

A key component of the SIR model concerns how likely it is for susceptible individuals to become infected. We follow the standard approach, according to which their probability of infection depends on the mass of already infected individuals μ_t^I and effective social distancing measures θL_t. The mass of newly infected individuals is denoted by μ_t^x and we assume that it is determined by

$$\mu_t^x = \pi_x \left[(1-\theta L_t)\mu_t^I\right] \left[(1-\theta L_t)\mu_t^S\right]. \tag{6}$$

The presence of $1 - \theta L_t$ twice in the above expression reflects the fact that lockdowns reduce the social interactions of both the infected and susceptible. The parameter π_x captures the degree to which the disease is contagious.[5] The mass of susceptible individuals in period $t+1$ is that of period t net of any new infections,

$$\mu_{t+1}^S = \mu_t^S - \mu_t^x. \tag{7}$$

Infected individuals remain in this state with probability π_I. The mass of infected individuals in period

5. In the quantitative analysis in Section 4, we allow π_x to be time varying to better capture the lags in timing for mobility and infections in the data.

$t+1$ equals a π_I share of the infected in period t plus any new infections. The resulting law of motion is

$$\mu^I_{t+1} = \pi_I \mu^I_t + \mu^x_t. \tag{8}$$

With probability $1 - \pi_I$, each infected individual either recovers or dies. Like Alvarez, Argente, and Lippi (2020), we assume that the probability of dying from the disease conditional on being infected $\pi_D(\mu^I_t)$ depends on the measure of current infections, resulting in $\phi_{D,t} = \pi_D(\mu^I_t)\mu^I_t$ fatalities every period. We assume $\pi'_D(\mu^I_t) > 0$ to capture the role of health care capacity for the fatality rate; a large number of infections puts a strain on the health care system, hurting its ability to successfully treat cases. The resolution of infections into recoveries or deaths induces the following laws of motion for these last two groups:

$$\mu^R_{t+1} = \mu^R_t + \left[1 - \pi_I - \pi_D(\mu^I_t)\right] \mu^I_t, \tag{9}$$

$$\mu^D_{t+1} = \mu^D_t + \pi_D(\mu^I_t)\mu^I_t. \tag{10}$$

The epidemic induces a law of motion for the overall population size N_t,

$$N_{t+1} = \mu^S_{t+1} + \mu^I_{t+1} + \mu^R_{t+1}. \tag{11}$$

As is well known from the epidemiological literature, in such a SIR model, the epidemic eventually winds down as the mass of infected individuals asymptotes to zero. Without social distancing measures, the SIR parameters π_x, π_I, and $\pi_D(\mu^I_t)$ determine the duration and severity of the outbreak. Social distancing policies L_t can alter these outcomes. In practice, we adopt an assumption that the epidemic ends H periods after it starts because a vaccine becomes available. With the discovery of a vaccine, all susceptible individuals are vaccinated and become functionally recovered, so no new infections can occur. This introduces a natural and numerically convenient terminal condition for our analysis, but H can be arbitrarily large.

2.4 The Government's Problem

The government and its international lenders learn about the epidemic in period 0. The outbreak changes the prospects for the economy, because the epidemic will lead to loss of life and disruptions in production. During the epidemic, we set up a centralized problem for the government, which makes all choices for this economy. It borrows from international financial markets, with an option to default, and chooses lockdown policies L_t to reduce the loss of life from the epidemic.[6] We study a Markov problem, which we solve backwards from the vaccine period H.

Consider first the government's problem for any period before the vaccine arrives $t < H$. The state variable for the government consists of the measures of each group $\mu_t = (\mu^S_t, \mu^I_t, \mu^R_t)$ and its debt B_t. The accumulated deaths are the residual, $\mu^D_t = 1 - \mu^S_t - \mu^I_t - \mu^R_t$. The value function for the government

6. We do not study whether households' own distancing measures would result in activity levels higher or lower than what is called for by our centralized solution. Farboodi, Jarosch, and Shimer (2020) show that in a similar model but without financial market frictions, government-mandated lockdowns improve over private choices, owing to negative externalities in contagion. These findings do not directly translate to our environment because debt crises bring additional negative externalities arising from, e.g., agents who do not internalize that by self-quarantining they are possibly worsening the debt crisis.

depends on these states and on time $V_t(\mu_t, B_t)$. The bond price function depends on future states and time, $q_t(\mu_{t+1}, B_{t+1})$, because default decisions will depend on these variables.[7] The government takes as given future value functions $V_{t+1}(\mu_{t+1}, B_{t+1})$ and the bond price schedule. It chooses optimal borrowing ℓ_t, partial default d_t, and lockdowns L_t to maximize its objective, given by

$$V_t(\mu_t, B_t) = \max_{\ell_t,\, d_t \in [0,1],\, L_t \in [0,1]} [u(c_t) - \chi \phi_{D,t}] + \beta V_{t+1}(\mu_{t+1}(\mu_t, L_t), B_{t+1}), \tag{12}$$

subject to the constraint

$$N_t c_t + (1 - d_t)(\delta + r) B_t = \tilde{z}\gamma(d_t)[N_t(1 - L_t)]^\alpha + q_t(\mu_{t+1}(\mu_t, L_t), B_{t+1})\ell_t; \tag{13}$$

the evolution of debt (4); the SIR laws of motion (6)-(9), which map current population measures and lockdown policies to future measures $\mu_{t+1}(\mu_t, L_t)$; fatalities induced by these dynamics $\phi_{D,t} = \pi_D(\mu_t^I)\mu_t^I$; and the total population constraint (11). The government internalizes that its choices for debt and lockdown affect the states tomorrow and the bond price.

When choosing L_t, the government trades off the potential benefits from saving lives against the costs of lockdowns in terms of output and consumption. Consumption is lowered by output disruptions from social distancing measures, and this response is amplified by the limited availability of international credit as well as the presence of default risk. If financing opportunities are ample and default risk is low, output disruptions matter for consumption only through a reduction of lifetime income. Consumption would then adjust to the lower permanent income, but the period-by-period consumption decline need not necessarily mirror the contemporaneous declines in output from lockdowns.

The government's problem results in decision rules for economic variables in periods $t = 0, 1, \ldots, H - 1$ for government debt $B_{t+1} = \mathbf{B_{t+1}}(\mu_t, B_t)$, default $d_t = \mathbf{d_t}(\mu_t, B_t)$, lockdowns $L_t = \mathbf{L_t}(\mu_t, B_t)$, and per capita consumption $c_t = \mathbf{c_t}(\mu_t, B_t)$. The problem also induces policy functions for the evolution of epidemiological variables that depend on the level of debt as well as the distribution of the population over types. Debt affects epidemiological dynamics through its impact on lockdowns. Let the equilibrium policy functions for the evolution of measures of susceptible, infected, and recovered individuals be $\boldsymbol{\mu}_{\mathbf{t+1}}(\mu_t, B_t)$.

The bond price schedule $q_t(\mu_{t+1}(\mu_t, L_t), B_{t+1})$ satisfies

$$q_t(\mu_{t+1}(\mu_t, L_t), B_{t+1}) = \frac{1}{1+r}\left\{(\delta + r)(1 - \mathbf{d_{t+1}}) + [1 - \delta + \kappa(\delta + r)\mathbf{d_{t+1}}]\, q_{t+1}(\boldsymbol{\mu}_{\mathbf{t+2}}, \mathbf{B_{t+2}})\right\},$$

where future default, borrowing, and lockdowns are given by equilibrium policy rules from the government problem, and they are taken as given at time t by the government—a Markov environment. The problem from period H onward is similar to the setup described above, except that the vaccine at period H moves all the susceptible agents to the recovered state and resolves all infections. Appendix B provides a definition of the equilibrium.

7. The time dependency of these functions, captured by the t subscript, reflects the time horizon to the vaccine. They would be time invariant without a terminal condition. The results of the baseline quantitative analysis are not sensitive to the timing of vaccination, as long as it is far enough into the future, as illustrated in Section 4.6.

3 Interactions between the Health and Debt Crises

In this section we simplify our model to characterize analytically the interactions between the health and debt crises. We establish that the epidemic increases default risk, which in turn worsens the epidemic. Social distancing and lockdowns work as investments in lives, and default risk limits the economy's ability to tap future resources, resulting in inefficiently low investment—an inefficient lockdown.[8]

The simplified model has only two periods. The economy starts without any debt $B_0 = 0$ and with initial measures of susceptible μ_0^S, infected μ_0^I, and recovered μ_0^R individuals. The value of the government is over consumption and life, $[u(c_0) - \chi\phi_{D,0}] + \beta[u(c_1) - \chi\phi_{D,1}]$. In period 0, the government chooses lockdowns L_0, borrowing B_1, and consumption c_0. In period 1, it chooses default d_1 and consumption c_1.

The lockdown L_0 reduces new infections in period 0, thereby reducing deaths in period 1. Specifically, we can write period 1's deaths as $\phi_{D,1}(L_0) = \pi_D(\mu_1^I(L_0))\mu_1^I(L_0)$, with the infected mass coming from both the unresolved initial infections and the newly infected, $\mu_1^I(L_0) = \pi_I\mu_0^I + \pi_x(1-\theta L_0)^2\mu_0^I\mu_0^S$. We assume $\phi_{D,1}(L_0)$ is decreasing and convex in lockdowns L_0, $\partial\phi_{D,1}/\partial L_0 \leq 0$ and $\partial\phi_{D,1}^2/\partial L_0^2 \leq 0$. Since the government cannot alter deaths in period 0, $\phi_{D,0}$, or population in period 1, N_1, as both are pinned down by the initial level of infection and epidemiology parameters, we assume for simplicity zero initial deaths, $\phi_{D,0} = 0$. For a cleaner exposition, we also restrict attention to $\theta = 1$ and linear production, $\alpha = 1$. Appendix D lays out the details of this stripped-down version of our model and the assumptions that guarantee an interior solution for partial default d_1 and lockdown L_0. In particular, we require that the output in default $\tilde{z}\gamma(d)$ is differentiable, decreasing, and concave in d.

We construct the solution to the government's problem working backwards. In period 1, the government cannot borrow and only chooses partial default to weigh its marginal benefit and cost, $-\tilde{z}\gamma'(d_1) = B_1$, where $-\tilde{z}\gamma'(d_1)$ reflects the marginal cost of a higher default intensity in terms of lost production and B_1 is the marginal benefit of lowering repayment. Let the optimal default decision be $d_1(B_1) = (\gamma')^{-1}(-B_1/\tilde{z})$, with $(\gamma')^{-1}$ the inverse of the derivative of γ. Under the assumption that $\gamma(d_1)$ is concave, partial default d_1 increases with B_1. The bond price schedule in period 0 reflects these default incentives, satisfying $q_0(B_1) = \dfrac{1}{1+r}(1 - d_1(B_1))$.

In period 0, the government chooses borrowing B_1 and lockdowns L_0. We derive a standard optimality condition for the borrowing choice as

$$u'(c_0) = \beta(1 + r^d(B_1))u'(c_1),$$

where $1 + r^d(B_1)$ is the *domestic interest rate*. It depends on the risk-free rate and on the elasticity of the bond price schedule with respect to borrowing $1 + r^d(B_1) = (1+r)/(1-\eta(B_1))$, where $\eta(B_1) = -\partial\ln(q_0)/\partial\ln(B_1) \geq 0$. The domestic interest rate reflects the shadow cost of borrowing and with default risk, it is higher than the risk-free rate, $r^d(B_1) \geq r$. The consumption in period 0 is then a fraction of lifetime income that decreases with the domestic interest rate, so that

$$c_0 = \frac{1}{1 + \frac{1}{1+r}[\beta(1+r^d(B_1))]^{1/\sigma}}\left(\tilde{z}(1-L_0) + \frac{1}{1+r}\tilde{z}\gamma(d_1)\right), \tag{14}$$

8. This mechanism is linked to the literature emphasizing the impact of limited enforcement on under-investment, example of which include Thomas and Worrall (1994) and Aguiar and Amador (2011).

where period 0 income $\tilde{z}(1 - L_0)$ depends on the lockdown policy and period-1 income is shaped by the default cost $\tilde{z}\gamma(d_1)$ and is discounted at the risk free rate $1 + r$.

The optimal lockdown L_0 equates the marginal cost of reducing current consumption c_0 to the marginal benefit of saving lives in period 1. Future default risk d_1 reduces lifetime income and tends to reduce current consumption c_0, which in turn increases the cost of lockdowns. We capture these forces with the following optimality condition:

$$\tilde{z}u'(c_0) = \beta\chi \left(-\frac{\partial \phi_{D,1}(L_0)}{\partial L_0} \right). \tag{15}$$

The left-hand side of (15) is the marginal cost of lockdowns in terms of reducing current consumption. For one extra unit of lockdown, output drops by $\tilde{z}$ units, which are worth $\tilde{z}u'(c_0)$. The right-hand side represents the marginal value of lockdowns in terms of saving lives. One extra unit of lockdown reduces deaths by $(-\partial\phi_{D,1}/\partial L_0)$, which is worth $\chi(-\partial\phi_{D,1}/\partial L_0) \geq 0$, given that χ is the value of one life.

With the following propositions, we establish that the health and debt crises reinforce each other. We first show that the pandemic leads to a higher default risk in period 1.

Proposition 1 (The epidemic generate default risk)**.** *The default intensity d_1 increases following an epidemic outbreak in period 0.*

See Appendix D for the proof. The epidemic generates default risk because a desire to smooth consumption induces higher borrowing and thus a higher default risk in the future. The increased default risk worsens financial conditions in period 0. In our general model, there are two additional mechanisms that lead to more default. First, following an unexpected epidemic outbreak in the first period, lockdowns lower the marginal cost of defaulting, and partial default increases. Second, lenders internalize poor future prospect which tighten the bond price schedule for long-term debt. High borrowing rates further increase higher default incentives.

Next, we illustrate our second point: future default risk reduces lockdown incentives and worsens the epidemic. To show this, we introduce a reference model with perfect financial markets, in which the government commits to fully repaying its debt; that is, $d_1 = 0$. We establish that the lockdown intensity in our baseline model is lower than the efficient level from the setup with perfect financial markets. With limited enforcement, the economy tends to "under-invest" in life-saving measures and ends with too many deaths.

With full commitment, the government chooses consumption and lockdown to maximize its value, subject to the evolution of the epidemic and a lifetime budget constraint. The optimal lockdown in this commitment setup also satisfies equation (15). Perfect financial markets, however, allow the country to smooth consumption across time and support a higher level of consumption in period 0, c_0^e. Specifically, consumption in period 0, c_0^e, satisfies

$$c_0^e = \frac{1}{1 + \frac{1}{1+r}[\beta(1+r)]^{1/\sigma}} \left(\tilde{z}(1 - L_0^e) + \frac{1}{1+r}\tilde{z} \right), \tag{16}$$

where L_0^e is the lockdown in this case. For a given lockdown L_0, consumption in period 0 is higher with perfect financial markets for two reasons. First, permanent income under perfect financial markets is higher than in the baseline model because of the absence of default costs. Second, the share of permanent income allocated for c_0 is also higher because the domestic interest rate is given by the risk-free rate r, which is lower than the one with default risk, $r^d(B_1)$. Increased consumption reduces the cost of lockdowns and generates a more intense lockdown in the perfect financial markets case.

Proposition 2 (Default risk worsens the epidemic)**.** *Deaths are higher with default risk than in an economy with perfect financial markets.*

See Appendix D for the proof. With default risk, the consumption cost of social distancing is higher, which results in lockdowns of lower intensity. Less mitigation elevates infections and results in more deaths.

4 Quantitative Analysis

We now turn to the quantitative analysis of the general model. Our goal is a quantitative evaluation of our mechanisms and policy counterfactuals. We first discuss the choice of parameters, including those controlling the SIR dynamics and the cost of default, and our moment-matching exercise with data from Latin America. We then describe the time paths of the economy and assess quantitatively the extent of the health, economic, and debt crises. To highlight financial markets' role for epidemic dynamics, we compare these time paths with those of two reference models, one with perfect financial markets and another with financial autarky. Finally, we conduct counterfactual debt relief experiments and show that these programs can deliver large social gains.

4.1 Parameterization and Data

We consider a weekly model to capture the fast dynamics of infection. We fix some of the parameters to values from the literature and estimate others with a moment-matching exercise to data from Latin America.

Epidemiological parameters. The SIR parameters are set based on findings in epidemiological research. According to Wang et al. (2020), the duration of illness is on average 18 days.[9] For our weekly model this implies a value for π_I, the parameter determining the rate at which infected individuals either recover or die from the disease, of $(1 - 1/18)^7 = 0.67$.

The parameter π_x relates to the widely used "reproduction number" $\mathcal{R}_0$, which measures the expected number of additional infections caused by one infected person over the entire course of his or her illness, with $\pi_x = (1 - \pi_I)\mathcal{R}_0$. As shown by Atkeson, Kopecky, and Zha (2020), the data on infections and deaths from COVID-19 are useful for recovering the underlying *effective* reproduction number for the epidemic, as non-pharmaceutical interventions, such as masking and testing, influence the degree to which the infection is transmitted and thus the value of this number. To that end, we assume that $\mathcal{R}_{0,t}$ is time varying because of health policies and behavioral responses. We impose a simple functional form controlled by the initial and final reproduction numbers $\{\mathcal{R}_{0,\text{ini}}, \mathcal{R}_{0,\text{end}}\}$ and a

9. This is also the value used by Atkeson (2020) and Eichenbaum, Rebelo, and Trabandt (2020).

decay rate ρ such that $\mathcal{R}_{0,t} = \mathcal{R}_{0,\text{ini}}\, \rho^t + \mathcal{R}_{0,\text{end}}(1-\rho^t)$. We set the $\mathcal{R}_{0,\text{ini}}$ to 2.6, based on early estimates of the reproduction number, including those from the *Diamond Princess* cruise ship, and estimate the other two parameters to Latin American data, as explained below.[10]

Following Alvarez, Argente, and Lippi (2020), we assume the case fatality rate $\pi_D(\mu_t^I)$ depends on the number of infected individuals to capture congestion effects in the healthcare system, so that $\pi_D(\mu_t^I) = \pi_D^0 + \pi_D^1 \mu_t^I$. The parameters π_D^0 and π_D^1 control the mortality of the infected. We assume that, in the absence of health care capacity constraints, the fatality rate is 0.5%, which is within the range of parameters used in recent papers studying COVID-19. Using this estimate, we set $\pi_D^0 = (1-\pi_I) \times 0.005 = 0.0016$, and we estimate π_D^1 to fit the data. The lockdown effectiveness is set to $\theta = 0.5$, following estimates from Mossong et al. (2008), who study the role of social contacts for the spread of infectious diseases and report that about half of infections occur away from the workplace, school, or travel and leisure (i.e., largely within the home). Finally, we assume that a vaccine arrives two years after the outbreak of the epidemic, $H = 104$. The arrival of the vaccine turns out to be largely irrelevant in our baseline model because "herd immunity" is reached before this two-year mark. Section 4.6 reports a robustness check over the time horizon to the arrival of the vaccine.

Table 1: Parameterization

Parameters	Value	*Moments*
Assigned parameters		
Preference		
Intertemporal elasticity $1/\sigma$	0.5	Standard value
Discount factor β	0.9996	Domestic annual real rate 2%, emerging markets
Value of life χ	4025	Viscusi and Masterman (2017)
SIR and lockdown parameters		
Initial SIR contagion rate $\mathcal{R}_{0,\text{ini}}$	2.6	*Diamond Princess* estimate
SIR resolution π_I	0.33	Mean recovery 18 days
SIR fatality π_D^0	0.0016	Baseline fatality rate 0.5%
Lockdown effectiveness θ	0.5	Mossong et al. (2008)
Debt and default parameters		
Risk free rate r (annualized)	1%	International real rate of 1%, annual
Long-term debt decay δ	0.0037	Debt duration of 5 years
Debt recovery factor κ	0.54	Cruces and Trebesch (2013)
Default costs γ_0, γ_1	0.04, 1.62	Arellano, Mateos-Planas, and Ríos-Rull (2019)
Parameters for moment-matching		
Congestion π_D^1	1.65%	Daily deaths: peak level and timing
Reproduction No. dynamics $\{\mathcal{R}_{0,\text{end}}, \rho\}$	$\{1.2, 0.9\}$	Lockdowns: peak level and timing
Default cost γ_2	0.0014	Mean debt-to-GDP 60%

Preferences. Flow utility over consumption features a constant relative risk aversion, with a coefficient set to the standard value of 2. The discount factor β is set to match an average domestic real rate of 2% for emerging market inflation targeters, as reported in Arellano, Bai, and Mihalache (2020).

10. See, for example, the analysis in Zhang, Diao, Yu, Pei, Lin, and Chen (2020).

An important parameter for our model is the cost of losing a life, χ. This parameter relates to the *value of statistical life* (VSL), which measures the marginal willingness to take on mortality risk. Viscusi and Masterman (2017) report estimates of the VSL across countries in 2015, and in our calculation we use their estimate of 1.695 million for Brazil.[11] Using an annual interest rate of 2% and a residual life of 40 years, we can express the VSL in terms of a weekly flow of $1,200, which implies a willingness to pay of $1.2 (or 0.85% weekly consumption) for 0.1% of reduction in mortality risk.[12] We use this calculation to set χ as the solution to the following equation:

$$\frac{1-\beta^{10\times 52}}{1-\beta}u(1) - 0.001\chi = \frac{1-\beta^{10\times 52}}{1-\beta}u(1-0.0085),$$

where we assume that the representative COVID-19 fatality has 10 years of residual life. The implied value for χ is 4,025, given our parameter values for β and the coefficient of relative risk aversion.

Debt and default. We set the annual international risk-free rate to 1%, which is the average real rate for U.S. Treasury bills since 1985. We pick δ to induce an average Macaulay debt duration of five years, in line with estimates for emerging markets. As in Arellano, Mateos-Planas, and Ríos-Rull (2019), we assume that the default cost is a concave function of the default intensity, $\gamma(d) = [1-\gamma_0 d^{\gamma_1}](1-\gamma_2 \mathbb{1}_{d>0})$, where the indicator $\mathbb{1}_{d>0}$ is 1 if d is positive so that a share γ_2 of productivity is lost if the country defaults at all, with any intensity. We adopt estimates for γ_0 and γ_1 from Arellano, Mateos-Planas, and Ríos-Rull (2019) and estimate the fixed cost parameter γ_2. The debt recovery κ is set to 0.54, consistent with the evidence in Cruces and Trebesch (2013), once preemptive restructurings are excluded. We set $\tilde{z} = 1$, a normalization of pre-pandemic steady state output to 1. The top panel in Table 1 collects all assigned parameter values.

Data. We compile data from Latin America for eleven countries: Argentina, Brazil, Chile, Colombia, Ecuador, El Salvador, the Dominican Republic, Mexico, Peru, Paraguay, and Uruguay. We collect time series during the year 2020 for deaths from the COVID-19 epidemic, Google Mobility data, government debt, and sovereign spreads.

The measure for deaths is average daily deaths per 10,000 people, monthly for the year 2020. This information is taken from the worldometers.info (2021) website. The Google Mobility measure we use is the weekly decline in workplace traffic[13] relative to January 2020, provided by Google LLC (2021). We compare our model with the average data series from these countries at a monthly frequency. In constructing these time series, we first time-aggregate weekly observations into monthly series and then average these monthly series across countries, weighting each country's value by its population in 2019.

11. In practice, the VSL estimates of Viscusi and Masterman (2017) do not vary much among countries. In terms of annual consumption per capita, they find that the VSL is 184 for Argentina, 229 for Brazil, 224 for Mexico, and 207 for the U.S.

12. To calculate the weekly flow, we solve for x in

$$1.695 \times 10^6 = \frac{1-0.9996^{40\times 52}}{1-0.9996}x,$$

which implies $x = 1,200$. The consumption data is in terms of 2015 U.S. dollar, from the World Bank.

13. Workplace traffic data imply an upper bound on the reduction in activity, as some work could be done from home. Nevertheless, Dingel and Neiman (2020) and Saltiel (2020) find that in developing countries only a minority of jobs can be performed at home: at most 25%, but as low as 10% for Colombia and Ecuador.

The data for government debt are taken from local primary sources such as each country's central bank, statistics office, or finance ministries. We compile quarterly data for 2019 and 2020 and report debt relative to gross domestic product. To set the initial debt level, we use the data in the fourth quarter of 2019 averaged across countries, weighting by population in 2019. The data for sovereign spreads are the EMBI+ series for all the countries for which they re available at a monthly frequency for the year 2020. The data are from Global Financial Data (2021). We average across countries, again weighting by population, and report the resulting time series relative to its value in January 2020.

The first column of Table 2 reports select moments for the data. The COVID-19 epidemic has hit Latin American countries quite hard, as seen in the data on daily deaths and lockdowns. Daily deaths peaked at 0.047 per 10,000 people during July 2020 and were 0.03 on average for the year. Google Mobility data imply that workplace hours were reduced by 48% at the trough during April 2020 and on average by 21%. The table also reports elevated sovereign spreads, peaking at 5.5% above their January level. The table finally shows that government debt to output across these countries was on average 60% at the onset of the epidemic.

4.2 Model Fit

We now describe our moment-matching exercise and the fit of the model. We estimate four parameters of our structural model: two controlling the dynamics of the epidemiological reproduction number, $\mathcal{R}_{\text{end}}$ and ρ; the fatality parameter π_D^1; and the fixed cost of default γ_2. We do so by targeting five moments of the data: the peak and timing of the daily deaths, the peak and timing of lockdowns, and the initial government debt to output ratio.

We interpret the data as being in steady state during January of 2020. In a pre-pandemic steady state, the model features an endogenous debt to output ratio, which we fit by choosing the fixed cost of default. We start the epidemic in the model in the first week of April 2020 by introducing a small fraction of infected individuals into the population, $\mu_0^I = 0.5\%$, and 3% of these already recovered or immune to the disease, $\mu_0^R = 3\%$. We then compute and simulate the model, to calculate the relevant time paths. We compare the model with our daily deaths data by multiplying the model weekly deaths per population, $\pi_D(\mu_t^I)\mu_t^I$, by $10,000/7$ to scale it to the relevant daily number and average across four weeks. We compare lockdowns in our model directly with the Google Mobility measure.

Appendix E describes our computational algorithm. To summarize it, we first compute the model without an epidemic. This serves as an initial condition and provides the basis of the economy after the epidemic. We then compute our benchmark model backwards, starting from the terminal period H when the vaccine arrives. As shown in the Appendix, the period H problem is very similar to the pre-epidemic problem, as no new infections occur. The solution consists of time-dependent bond prices, policy, and value functions. We then simulate forwards given our initial conditions, to construct the economy's paths during the epidemic.

Table 2 presents the model fit. In our model, daily death peak at 0.047 in July 2020, which matches the data. The model also generates a sizable lockdown of 41% during April 2020, close to the corresponding data value. Debt to output also matches well the value observed in the data. An important aspect of our parameterization, which enables this fit, is the time-varying reproduction number $\mathcal{R}_{0,t}$, especially with respect to the asynchronicity between the peaks of daily deaths and lockdowns. The model rationalizes

Table 2: Model Fit

	Data	Model
Targeted moments		
Daily deaths (per 10K)		
Peak	0.047	0.047
Timing of peak	Jul-20	Jul-20
Lockdown intensity (%)		
Peak	48	41
Timing of peak	Apr-20	Apr-20
Initial debt-to-output	60%	60%
Out of sample moments		
Average daily deaths	0.030	0.033
Average lockdown intensity	21	25
Spreads		
Peak	5.5	5.9
Timing of peak	Apr-20	Apr-20
Average	2.2	4.9

the intense lockdowns in April as a response to a very high initial reproduction number. The high daily deaths in late July, in turn, are the result of the epidemic running its course under lockdowns, as well as the congestion effects captured by π_D^1, even though the effective infectiousness of the disease is already lower. The timing and level of the peak in daily deaths also reflect the decay rate and terminal level of the reproduction number, both set endogenously. In Section 4.6, we consider alternative dynamics for the reproduction number.

Table 2 also includes salient out-of-sample moments. In the model, the average daily deaths and lockdowns for 2020 are 0.033 and 25%, respectively, close to the data counterparts of 0.03 and 21%. For the debt crisis, we find that sovereign spreads spiked early in the epidemic, in both the model and data. Spreads peaked at 5.9% in April in the model, close to the data spike of 5.5%, also in April. The model predicts, however, a more persistent rise in spreads. The average government spread from March to December in the model is 4.9%, while in the data, this number is 2.2%. In interpreting the more persistent spreads of the model, it is useful to note that our baseline abstracts from any financial assistance from third parties. As we describe below in Section 4.5, many countries received substantial financial assistance in 2020, and in line with our analysis of such programs, this helped dampen the increase of spreads during the epidemic.

4.3 Baseline Dynamics

We now describe the dynamics of all key variables during the epidemic. Recall that the economy is hit by the epidemic in the first week of April 2020, when the government has an outstanding debt given by the steady-state level in the absence of an outbreak. Figure 1 plots the time paths of epidemiological and economic variables of interest: the fraction of population that is deceased, infected, or susceptible;

the intensity of social distancing measures; government debt and spreads; consumption; and output. The paths run through January 2023, and the vertical lines in the plots represent the date at which the vaccine arrives.

Panel (a) of Figure 1 plots the evolution of the deceased μ_t^D. Our model predicts that the eventual death toll from the epidemic is 0.16% of the population, which corresponds to 840,000 people for the Latin American countries in our data, with a total population of 525 million in 2019. Panel (b) plots lockdowns and shows that they start promptly upon the outbreak, remain at a 40% level for about two months, gradually wind down, and last about a year. These social distancing measures reduce the death toll of the epidemic. At our parameter estimates, if we were to impose no social distancing measures during the epidemic, the SIR laws of motion would predict a death toll of 1.23% of the population, or 6.5 million people in the Latin American countries of our sample.

Figure 1's panels (c) and (d) plot the evolution of infected and susceptible measures during the episode. The infected portion of the population reaches its peak of 1.2% in July 2020. The fraction of susceptible individuals falls smoothly as the epidemic progresses, until about 75% of the population is still vulnerable to infection. After two years the vaccine arrives and all the remaining susceptible individuals become immune, but the vaccine comes late, as the brunt of infections have passed and the country is close to "herd immunity."

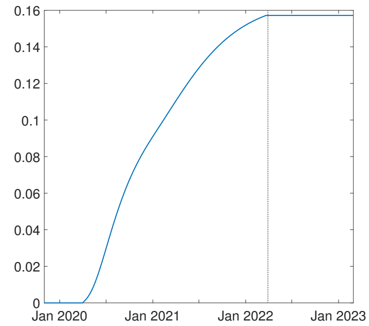
Panels (e) and (f) show the paths of sovereign spreads and government debt scaled by output before the epidemic. Spreads jump on impact by about 6% and decrease smoothly thereafter. They increase because the epidemic is unexpected and increases default risk. Government debt grows because additional borrowing is useful to support consumption and also because the government partially defaults on the debt, with defaulted payments accumulating. Debt to output increases until January 2021, when it reaches a peak of about 70% of initial output. The 10% increase in debt to output resembles the experiences of Latin American countries, which experience an increase in debt to output of 11% on average.[14] Afterwards debt falls, but quite slowly, as the economy converges to steady state after January 2023, about a year after the epidemic resolves. The persistently high level of debt during the epidemic leads to a prolonged period of elevated sovereign spreads and partial default.

Finally, panels (g) and (h) in Figure 1 show the paths for output and consumption. Output falls significantly at the outset of the epidemic because of the tight lockdowns. During 2020, output falls in the model by about 18%. Consumption falls too, by about 6.5%. The consumption decline is smaller than that of output because government borrowing and default support consumption. During 2021 and 2022, levels of consumption and output continue to be lower than they are in the steady state; they are about 5% below their pre-epidemic levels. Resources are fewer because of the default costs associated with the protracted debt crisis. The economy approaches the steady state by early 2023.

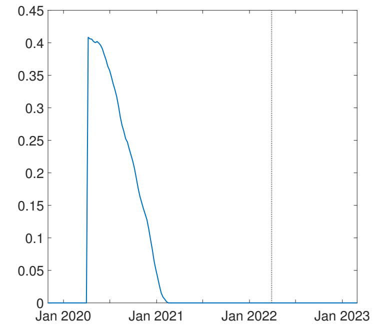
These time paths suggest that the epidemic creates a combined health, debt, and economic crisis with scarring effect on output, consumption, and sovereign spreads that are more persistent than the health crisis itself.

14. Although we do not have data for all countries in our sample, we have used the available data for Brazil, Ecuador, Mexico, Paraguay, and Peru, in computing this average.

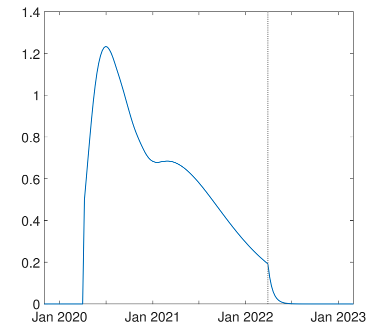
Figure 1: Dynamics in Baseline Model



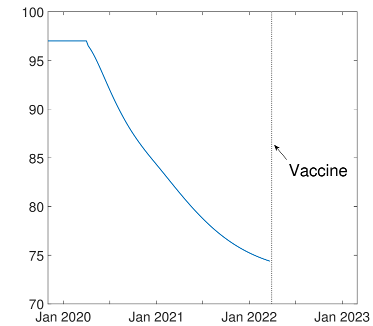
(a) Deceased



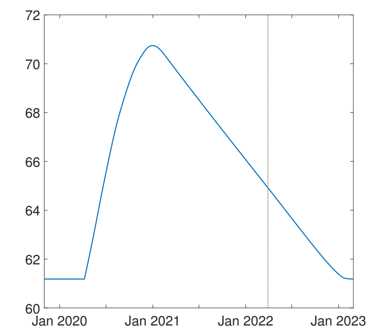
(b) Lockdowns



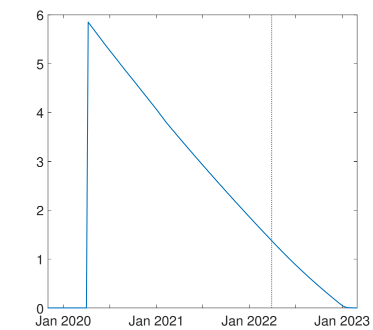
(c) Infected



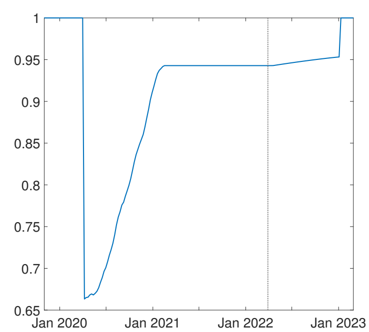
(d) Susceptible



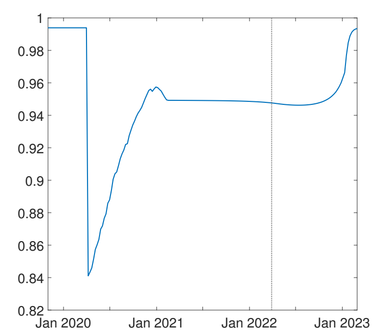
(e) Debt



(f) Spread



(g) Output



(h) Consumption

Notes: Deceased, Infected, and Susceptible are expressed in percentage of population. Debt is relative to pre-pandemic output. Spread is in percentage, relative to January 2020 value. Output and consumption are relative to pre-pandemic output.

Table 3: Epidemic Outcomes and Financial Markets

		Baseline with default	Perfect financial mkts	Financial autarky
Health crisis	Deceased (% population)	0.16	0.05	0.19
Economic crisis	Lockdown			
	Length (months)	11	16	10
	Intensity, max (%)	41	79	38
	Output loss (%)	−30	−27	−13
	Debt Increase (%)	15	70	0
Debt crisis	Length episode (years)	3	–	–
	Spread, max (%)	5.9	–	–
Welfare losses	Country CE	−28	−19	−30
(% output)	Lender	−7	–	–

Notes: Deceased is total epidemic deaths, percentage of initial population. Lockdown length is the number of months with positive values and its max intensity is a monthly value. Output losses are present discounted cumulative losses during the epidemic relative to the pre-epidemic annual output. Debt increase is the maximum increase in government debt relative to pre-epidemic annual output. Welfare losses for the country are reported as consumption equivalence (CE) present value measures relative to pre-epidemic measures (equation (18)) in units of output. Welfare losses for the lender are the value of initial debt relative to pre-epidemic values, in units of annual pre-epidemic annual output (equation (21)).

4.4 Epidemic Outcomes and Financial Markets

We now evaluate the role of financial market frictions for these dynamics. We compare our baseline economy with default risk against two counterfactual economies with varying degrees of financial market frictions: an economy with perfect financial markets and an economy in financial autarky.

Table 3 reports key moments for the health, economic, and debt crisis, as well as measures of the welfare cost of the pandemic for the baseline and counterfactual economies. We consider as a measure for the health crisis the total eventual deaths as a percentage of the initial population. For the economic crisis, we report the length and intensity of lockdowns, cumulative output losses, and the increase in government debt. The length of the lockdown is the number of months with strictly positive lockdown intensity $L_t > 0$, while the intensity of lockdowns is measured by its maximum during the event. The cumulative output loss is the present discounted value of the output path relative to the counterfactual output path without an epidemic, discounted at the risk-free rate and expressed in units of pre-epidemic annual output. The increase in sovereign debt is measured relative to its level in the absence of the epidemic and reported relative to pre-epidemic output. For the debt crisis, we focus on the length of the crisis, measured by the number of months with strictly positive partial default $d_t > 0$, and the government spread, as measured by its highest value during the event.

The table also reports welfare losses from the epidemic for both the country and its international lenders. The country suffers from the epidemic because of the loss of life and the declines in consumption that result from the epidemic and the debt crisis. To evaluate the country's welfare losses, we consider a consumption equivalence measure $c^{\text{eq}}(\mu_0, B_0)$ at the outbreak of the epidemic, implicitly defined by

$$\frac{1}{1-\beta} u(c^{\text{eq}}(\mu_0, B_0)) = V_0(\mu_0, B_0), \tag{17}$$

where $V_0(\mu_0, B_0)$ is the value function at time 0, at the outbreak of the epidemic, that is, the first week of April 2020. The value function V_0 reflects both the stream of consumption and the stream of deaths; our consumption equivalence measure summarizes these two streams into one value, which is the constant consumption flow that equals this value in the absence of any mortality risk. Using the country's discount β, we express the welfare loss as the present value of the consumption equivalence losses:

$$\text{CE present value} = \frac{c^{\text{eq}}(\mu_0, B_0) - c^{\text{pre,eq}}(B_0)}{1-\beta}, \tag{18}$$

where $c^{\text{pre,eq}}(B_0)$ is the pre-epidemic consumption equivalence when debt is B_0.

Lenders also suffer losses because the epidemic triggers a debt crisis that they did not anticipate, a drop in the value of the bonds they hold, effectively a capital loss. We report these losses as the change in the value of initial debt B_0 for the bond holders, which depends on the market price of this debt upon the outset of the epidemic, $\tilde{q}(\mu_0, B_0)$:

$$\text{Lenders' value}(B_0) = \tilde{q}(\mu_0, B_0) B_0. \tag{19}$$

This bond price depends on the structure of our long-term debt contracts, as well as current and future defaults. Recall that every unit of outstanding debt decays at rate δ, calls for a payment of $(\delta + r)$ every period without default, and accumulates at rate κ per defaulted unit when $d_t > 0$. In addition, note that the bond price function at time 0, $q_0(\mu_1(\mu_0, L_0), B_1)$, as specified in equation (5), compensates for the expected default losses from period 1 onward. This implies that the effective price of the outstanding debt upon the onset of the epidemic, before any decisions, is equal to

$$\tilde{q}(\mu_0, B_0) = (1 - d_0(\mu_0, B_0))(\delta + r) + [1 - \delta + \kappa(\delta + r) d_0(\mu_0, B_0)] q_0(\boldsymbol{\mu_1}, \mathbf{B_1}), \tag{20}$$

where the states $\{\boldsymbol{\mu_1}, \boldsymbol{B_1}\}$ are given by the government's decision rules, given initial conditions $\{\mu_0, B_0\}$. We report welfare losses for lenders as the change in their value relative to the pre-epidemic value:

$$\text{Lenders' loss} = \tilde{q}(\mu_0, B_0) B_0 - q^{\text{pre}}(B_0) B_0, \tag{21}$$

where $q^{\text{pre}}(B_0)$ is the bond price before the epidemic. We report losses in units of pre-epidemic annual output.

Baseline economy. Consider again the outcomes in the baseline model with default risk, captured by the first column of Table 3. These statistics summarize the time paths presented in Figure 1. The epidemic results in a final death toll of 0.16% of the population, and lockdowns are in effect for 11 months, with a peak intensity of 41%. The output losses from the epidemic equal 30% in present value; about half of the losses are due to lockdowns, and the other half are due to default costs. The government debt to output ratio increases by 10 percentage points to support consumption. The debt crisis lasts about four years, with elevated spreads and partial default. Table 3 reports that the welfare losses from the epidemic are significant: for the country 28% in terms of pre-epidemic output, which corresponds to a 0.6% drop in consumption equivalence every period. Lenders are also worse off by about 7%, owing to unexpected capital losses. The burden of the epidemic falls largely on the country.

We now compare the results of our baseline economy with those of our two reference economies, which have varying financial market conditions. For these comparisons, we keep all the structural parameters fixed across economies. Further details on these reference models are in Appendix C.

Perfect financial markets. In the reference model with perfect financial markets, the economy can freely borrow at the risk-free rate to support consumption, and social distancing measures affect consumption only through their effect on the present value of output. The epidemic outcomes for an economy with perfect financial markets are presented in the second column of Table 3.

With access to perfect financial markets, the economy fares substantially better. Total deaths are sharply cut to less than a third, about 0.05% of the population. The time path of daily deaths exhibits a much lower peak at 0.01, compared with the baseline's peak of 0.047. Such a reduction in fatalities is due to the longer and more intense lockdowns. Social distancing measures in this economy last for 16 months and are at a higher intensity, with a maximum level of 79%.[15] The present value of output losses in this case is somewhat smaller than in the baseline; in this economy, it is equal to 27%. The main difference with the baseline is that the output losses in this model arise strictly because of lockdowns and not because of default costs, which means the output costs in this economy are directly linked to investment in saving lives. Government debt is used aggressively to support consumption and increases by 70%.[16] With perfect financial markets, the economy does not experience a debt crisis, with its associated partial default and elevated government spreads. The welfare costs from the epidemic continue to be quite significant for the country while it does not affect lenders. During the epidemic, welfare is 19% lower than it was in the pre-epidemic economy. Comparing this welfare cost with the 28% one in the baseline, suggests that default risk accounts for over a third of the cost of the epidemic in the baseline.

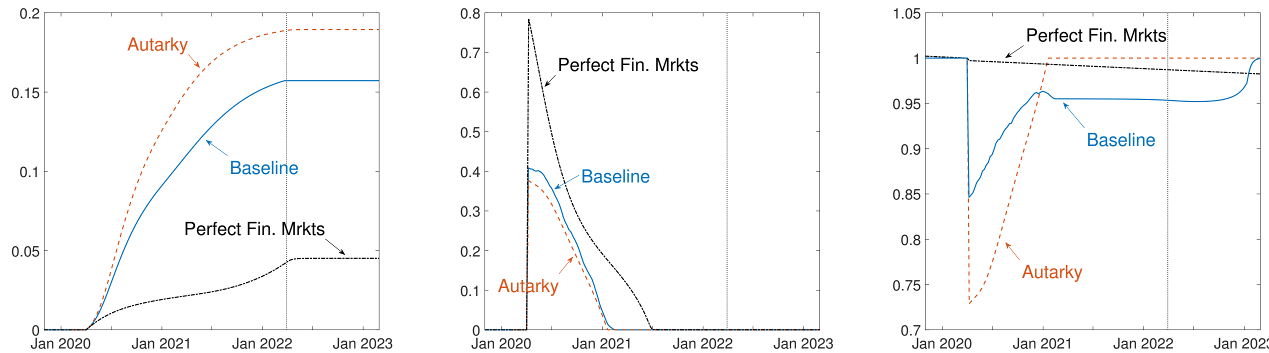
Financial autarky. We also consider outcomes in a counterfactual economy that experiences the epidemic without access to borrowing and remains in financial autarky throughout. Without market access, lockdowns affect consumption one-for-one and period-by-period and the epidemic results in worse outcomes. Without the possibility of external borrowing, the cost of social distancing is larger, leading to a choice of shorter and more limited lockdowns. The less stringent mitigation policies result in a higher death toll. The eventual death toll is 0.19% of the population, or about 20% more deaths than in the baseline. The output costs are more muted than in the baseline for two reasons. First, lockdowns are shorter with financial autarky, and second, the government does not experience any default costs. Finally, we find that the welfare costs from the epidemic are slightly higher under financial autarky

15. The decrease in fatalities with perfect financial markets goes a long way towards the minimum feasible fatalities from the epidemic, as discussed in Hethcote (2000) and the pedagogical exposition in Moll (2020).

16. With perfect financial markets, the economy has an incentive to increase borrowing even without the epidemic. To isolate the effect of the epidemic, we report the increase in debt relative to this preexisting trend.

because of the higher death toll and an inability to smooth consumption.

Figure 2: Epidemic Dynamics and Financial Markets



(a) Deceased (b) Lockdown (c) Consumption

Dynamics. Figure 2 compares the time paths of the deceased population, lockdowns, and consumption in our baseline model with those of two reference economies. The paths are plotted relative to those that would arise in the absence of the epidemic. The accumulation of deaths in panel (a) illustrates that access to better financial markets can be a powerful tool to dampen the costs of the health crisis and that sovereign default risk is literally deadly for an economy that faces an epidemic. These radically distinct outcomes for the evolution of deaths reflect the different lockdown policies implemented under these financial market conditions; such policies are presented in panel (b). The economy with perfect financial markets can support more stringent social distancing because it can use financial resources to support consumption. At the other extreme, financial autarky forces the government to pick less strict lockdowns, as the lost output translates one-to-one into lower current consumption.

The consumption paths for the three financial settings are compared in panel (c) of Figure 2. With perfect financial markets, consumption falls only to the extent called for by the decline in permanent income from the lockdowns. Consumption has a downward trend because the economy is more impatient than international lenders, as the parameters satisfy $\beta(1 + r) < 1$. Under financial autarky, consumption experiences the sharpest declines during lockdowns, but it recovers to pre-trend values as soon as lockdowns stop. In the baseline with default risk, the short run consumption decline is smaller than under financial autarky, but it is more persistent because of the lengthy debt crisis. The evolution of debt and default in the baseline worsens the consumption costs of the pandemic.

These quantitative findings echo the theoretical results in Section 3, in which we argued that default risk worsens the death toll of the epidemic and experiencing an epidemic outbreak will cause spreads to spike in turn. In contrast with the simple analytics of the two-period model of Section 3, we now emphasize the large magnitude of additional output losses and deaths caused by lack of commitment and the associated default risk. Ample access to credit and effective consumption smoothing reduce the cost of lockdowns and enable aggressive mitigation of the disease.

4.5 Debt Relief Counterfactuals During COVID-19

As we have shown, the structure of debt markets has a profound impact on epidemic outcomes, because the government's debt burden weighs heavily on the economy's ability to mitigate infection through social distancing. International financial assistance programs are therefore potentially useful policies to improve outcomes in emerging markets during the COVID-19 epidemic.

As mentioned earlier, the International Monetary Fund, the World Bank, the Inter-American Development Bank, and other international organizations have rapidly implemented debt relief programs to support countries during the COVID-19 epidemic. The IMF made available to countries about $250 billion in credit under programs such as the Catastrophe and Containment Relief Trust, the Rapid Credit Facility, and Standby Credit Facilities. The World Bank has worked with the G20 in extending debt relief to about 40 countries through the Debt Service Suspension Initiative and, through the Common Framework, is developing further restructuring guidance, for bilateral government debt with official creditors.

Motivated by these programs, we use our model to conduct two counterfactual experiments for debt relief. The first program we consider is a long-term, risk-free loan from a financial assistance entity. The second program is a voluntary restructuring program between the country and its private lenders. We evaluate how these programs change the outcomes of the epidemic, the debt and health crises, and their welfare properties. We find that these programs have a large positive social value because they shorten the debt crisis and allow for better mitigation policies which save lives. Next, we discuss the details of these programs.

Table 4: Debt Relief Counterfactuals

	Loan Program			Voluntary Restructuring
	50% Debt	60% Debt (baseline)	70% Debt	60% to 51%
Country welfare gains (% output)	5.4	7.5	7.4	10.6
Debt crisis: length reduction (years)	0.2	2	3	2
Debt crisis: max spread reduction (%)	0.4	4.2	7.5	4.1
Health crisis: deaths prevented (% deaths)	11.5	2.1	2.3	1.1
Lenders gains (% output)	0.4	4.7	6.6	0

Note: The loan program consists of a long-term, default-free loan equivalent to 10% of pre-epidemic output. We consider the effects of this program for an economy that starts with levels of debt of 50%, 60%, and 70%. The voluntary restructuring program reduces the debt of the country from 60% to 51%. Welfare gains for the country are the change of consumption equivalence in present value from the debt relief program (22); for lenders they are the change in value of initial debt (23). Reductions in the length of the debt crisis, the maximum spreads, and deaths are reported relative to the model without the programs.

A loan program. During the epidemic's outbreak, a financial assistance entity extends a default-free loan of fixed size to the country. The sovereign gets F in a lump-sum now and repays $\tilde{F}$ each period in perpetuity, starting after a grace period g. The financial assistance entity discounts the future at the international rate r and breaks even with this loan, so that the terms of the loan satisfy $\tilde{F} = r(1+r)^g F$. The problem of the government is identical to the one in the benchmark model, except for small alterations of the budget constraints. The constraint at the outbreak of the epidemic adds F to the resources on hand, while the budget constraints after the grace period subtract the payment $\tilde{F}$ from the available resources. We evaluate the effects of a loan size of 10% of pre-epidemic output, subject to a grace period of two years.

Table 4 presents the results of the counterfactual loan program, relative to those of the baseline. It focuses on the impact of the program for the debt crisis, as measured by the reduction in the length of the crisis and the maximum sovereign spread level, as well as the consequences for the health crisis, as measured by the percentage of deaths prevented. The table also reports the welfare gains to the country and its lender. These statistics are reported relative to relevant moments in the baseline economy, without any financial assistance program. As in the previous section, welfare gains and losses are expressed in present value of annual, pre-pandemic output units:

$$\text{Country welfare gain} = [c^{\text{eq, loan}}(\mu_0, B_0) - c^{\text{eq}}(\mu_0, B_0)]/(1-\beta) \tag{22}$$

$$\text{Lenders gain} = \tilde{q}^{\text{loan}}(\mu_0, B_0)B_0 - \tilde{q}(\mu_0, B_0)B_0, \tag{23}$$

where $c^{\text{eq, loan}}(\mu_0, B_0)$ is the consumption equivalence of the initial value under the loan program $V_0^{\text{eq, loan}}(\mu_0, B_0)$, and $\tilde{q}^{\text{loan}}$ is the equilibrium price of the outstanding debt under the loan program.

We consider the effects of the program on our baseline economy, which starts with a debt to output ratio of 60%, as well as the effects of this loan for less and more indebted economies. These economies are identical to our baseline economy in terms of parameters and the epidemic shock, but they have initial debt B_0 levels of 50% and 70% of output, respectively. Table 4 shows that the loan program generates considerable welfare benefits. For the baseline, the welfare gains are 7.5%. Recall that the epidemic leads to a decline in welfare of 28%; this loan program reduces the cost to 20.5%. These benefits arise because with the loan program, the debt crisis is shortened and spreads are lower. The default episode goes from three years to one year, a reduction of two years, while the maximum spread in the baseline is lowered from 5.9% to 1.6%, a reduction of 4.2%. The loan program benefits the country also because it improves health outcomes, as it reduces total deaths by 2.1% of the baseline number. The program also delivers gains to the private lenders that are holding the outstanding debt at the outbreak of the epidemic. The reduction in default losses from a smaller and shorter debt crisis increases the value of the debt for these bonds by 4.7% of pre-epidemic output. The overall social gain from the loan is a sizable 12.2%, as the country and its lenders gain, and the official lender breaks even.

The loan program can also benefit less and more indebted economies. Welfare gains are smaller for economies with less initial debt, at 5.4%, while they are a bit smaller also for more indebted economies, at 7.4%.[17] We find that, in general, the welfare gains from the loan program are non-monotonic with respect to initial debt level: how economies choose to deploy these new resources varies based on this initial indebtedness. Low debt economies face fewer difficulties in debt markets and hence use most of the loan to invest in saving lives by tightening lockdowns. The loan allows the economy to prevent 11.5% of the deaths from the baseline model. In contrast, more indebted economies use the loan program mostly to alleviate the debt crisis. The program allows them to reduce the length of the default crisis by three years and maximum spreads by 7.5%. The use of the loan program for investing in life-saving

17. The cost of the epidemic itself varies with the level of initial debt: the welfare cost of the epidemic is 26% and 29% for the economies with less and more debt, respectively.

lockdowns versus investing in better debt crisis outcomes shapes the non-monotonicity of welfare gains as a function of initial debt. In contrast, the gains for lenders from the loan programs are monotonically increasing in the level of debt; these gains increase from 0.4% to 6.6% when the economy's debt rise from 50% to 70%. The reason is that the extra resources obtained via the loan program are useful for reducing the costs of the debt crisis for more indebted economies, leading to higher capital gains for the initial lenders of such economies.

Our loan program relates to the work by Hatchondo, Martinez, and Onder (2017), who augment a standard long-term debt model with an option for the sovereign to take short-term, risk-free official loans up to an exogenous borrowing limit. They find that upon the surprise introduction of this official lending option, spreads decrease and country welfare improves. However, in the long run, the impact of this program is minimal because the sovereign endogenously adjusts its risky borrowing behavior. In our model without the epidemic, the loan program features effects similar to theirs. However, with the epidemic, our program has permanent effects because the reduction in death changes the outcome of the epidemic in the long run.

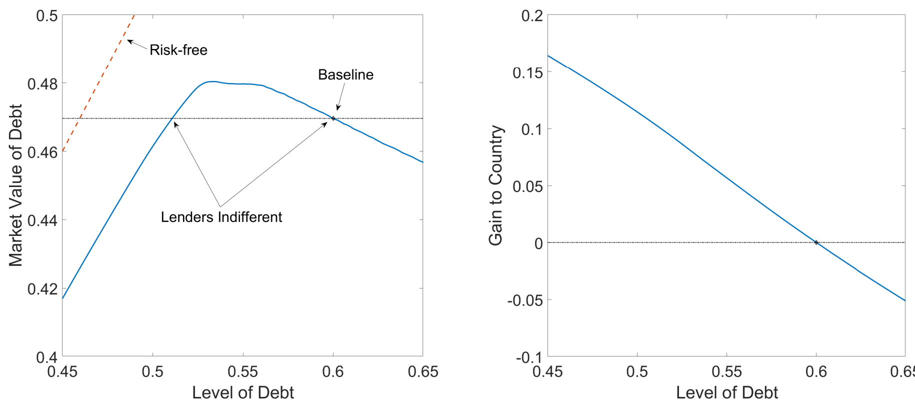
We conclude this section by comparing our results to the work on debt buybacks by Bulow and Rogoff (1988) and Aguiar, Amador, Hopenhayn, and Werning (2019). As in these papers, although the loan is not directed towards buying back the debt, in practice the economy uses it for this purpose. A large amount of resources received by the country during one week are used over time to adjust the level of debt. In contrast with these papers, however, we find large benefits to the country. One reason for this result is that in our economy, the loan can be used for *investment* in both lives and reducing default costs. The second reason is that we consider consumption equivalence measures under concave utility, and the loan program allows consumption smoothing while production is depressed from social distancing.

A voluntary restructuring program. We now evaluate the possibility of a voluntary restructuring program between the country and its creditors at the outset of the epidemic. Such a program occurs if both the country and its private lenders agree to a reduction in the initial level of debt from B_0 to B_0^{res}. A restructuring program is always beneficial for the country because its value $V_0(\mu_0, B_0)$ is everywhere weakly decreasing in debt. Their gains from debt reduction are always positive, as shown in panel (b) of Figure 3. The open question is whether private lenders that hold the outstanding debt would voluntarily agree to a reduction. In our quantitative analysis, we find that the answer is yes, because restructuring the debt sufficiently increases repayment capacity.[18]

Private lenders will agree to a voluntary reduction in debt only if their value weakly increases following such a program. As discussed in Section 4.4, the value to lenders of the outstanding debt $\tilde{q}(\mu_0, B_0)B_0$ depends on its effective price at the outset of the epidemic, $\tilde{q}(\mu_0, B_0)$, as defined in equation (20). The key property that makes a voluntary restructuring possible is that the price of the outstanding debt itself tends to fall with the level of this debt. The value of the outstanding debt, price times quantity, can increase from a reduction in the level of debt, the quantity, if the unit price increases fast enough, a

18. This reform relates to the voluntary exchange program of Hatchondo, Martinez, and Sosa-Padilla (2014), who randomly give the sovereign an opportunity to offer its lenders a voluntary restructuring program, with a size determined by Nash bargaining, in an otherwise standard long-term debt model. An alternative form of debt relief is considered by Hatchondo, Martinez, and Sosa-Padilla (2020), who evaluate debt standstills, a temporary suspension of debt service payments. Their overall assessment of this policy proposal is negative and strongly dominated by immediate haircuts.

Figure 3: Scope for Voluntary Restructuring of the Outstanding Debt



(a) Value to Lenders of Outstanding Debt (b) Gain to Country from Debt Reduction

property reminiscent of the "Laffer curve" for borrowing exhibited by models with sovereign default risk.

We can evaluate the scope for voluntary restructuring in our quantitative model by computing the lenders' value for the outstanding debt B_0, right as the pandemic starts. Panel (a) of Figure 3 plots this value as a function of the level of initial debt B_0, both axes scaled by the pre-pandemic steady state output level. It shows that the lenders' value increases with debt up until the level of debt is about 55%, flattens out, and decreases beyond that. The figure also shows that with default risk, the lenders' value for debt is quite depressed relative to its value in the case of risk-free debt, which is illustrated by the dash line. The dot represents the initial level of debt of 60% in our baseline economy. The fact that this initial level of debt is on the downward-sloping portion of the lenders' value curve gives room to voluntary restructuring: *holders of outstanding debt stand to gain from reductions in the level of debt.* The most ambitious voluntary restructuring to which lenders are willing to agree is a reduction of the country's debt to about 51% from the pre-pandemic steady state level of 60%. Any further reduction would make them strictly worse off, and they would not entertain it on a voluntary basis. We focus on the impact of this debt restructuring program for the country, as lenders neither lose nor gain value. As the right panel of the figure shows, the country gains about 10.6%, more than the approximately 9% reduction in debt. The last column in Table 4 shows that these large welfare gains for the country reflect significant improvements in the debt and associated crises. By implementing the program, the debt crisis is shortened by three years, spreads are lowered by 4.1%, and epidemic deaths are reduced by 1.1%.

4.6 Alternative Health Scenarios

We complete our analysis by evaluating alternative health scenarios and their impact on the crisis. We consider the consequences of a more infectious virus by comparing the dynamics of the baseline model with paths resulting from the same parameter values but faced with a) a constant high $\mathcal{R}_0$ in line with the initial conditions in the baseline, e.g., based on the upper bound of the *Diamond Princess* estimates in Figure 4, and b) a second wave of infections and deaths, as induced by the emergence of a new variant in early November 2020, in Figure 5. Finally, in Figure 6, we consider an alternative timing for the vaccine by pushing back its arrival by one additional year.

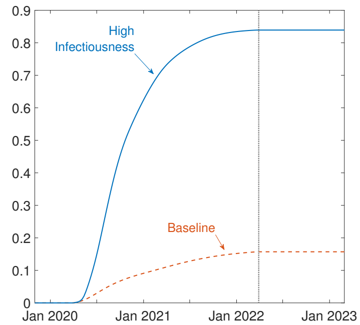
A highly infectious epidemic. Figure 4 documents the consequences of a permanently high level of infectiousness, with a fixed $\mathcal{R}_0$ of 2.6 throughout the pandemic. Under such a scenario, lockdowns are more intense and lengthier. Even so, the total number of eventual deaths is 4.5 times higher, with infections spiking to well above 4.5% of the population at the peak of the episode. The intense lockdowns call for a further substantial reduction in consumption and a run-up in debt to initial output of over 85%, with spreads jumping to about 12% and intense default over the following five years. Even though lockdowns are more intense, they are delayed and peak at the same time as infections. In preparation for the start of these delayed lockdowns, the government reduces debt to create additional fiscal space for itself. This pattern illustrates the role played by time-varying $\mathcal{R}_0$ in our baseline parameterization: to induce a spike in social distancing earlier than the peak of infections, as in the data.

A new variant and a second wave. As of the writing of this draft in early 2021, several countries in Latin America are experiencing a second large wave of COVID-19 cases arising from new COVID-19 variants. In this section, we illustrate how our model can be used to accommodate the possibility of such variants. We consider the emergence of a more infectious variant of the virus, leading to a "second wave" of infections and deaths. The new variant is substantially more contagious but not inherently more deadly, conditional on infection. It appears unexpectedly in early November and quickly becomes the predominant source of new infections. For tractability, we do not separately keep track of the number of infections with the new variant. Instead, we model its emergence as a spike in infectiousness by permanently increasing $\mathcal{R}_0$ to 1.6, eight months after the initial outbreak. We report the dynamics of this scenario in Figure 5.

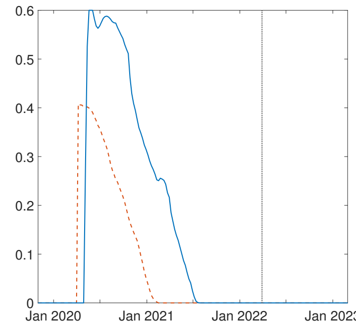
The impact of this new variant is dramatic. A second wave of elevated daily deaths arises, spiking to 0.07 in early February 2021. At that time, over 1.6% of the population is simultaneously infected. Eventually, total deaths are slightly under 0.26% of the initial population, more than 160% of the deaths in the baseline scenario with a single outbreak. The mitigation response calls for a second lockdown, which is lengthier but less intense, as seen in panel (b) of Figure 5. This results in a 25% drop in output at the trough and a 14% decrease in consumption shown in panels (g) and (h), respectively. Even so, the prospects of an impending vaccine induce a winding-down of social distancing measures by late 2021, leading to a smaller third uptick in infections and deaths as shown in panel (d). The drop in consumption is cushioned by additional borrowing as shown in panel (e), with debt reaching over 76%. Spreads spike anew to 9.2%, as lenders price in the additional borrowing and a more protracted debt crisis. As in the baseline, if we assume no international financial assistance and otherwise normal conditions in world financial markets, the severity of the second wave translates into a deep second wave of a protracted debt crisis.

Delayed vaccine availability. Emerging markets face the additional hurdle of limited medical and scientific resources for managing the epidemic. As vaccines and treatments become available, accessing these could lead to additional expenses and constraints. Our model suggests, however, that if these innovations become available far enough into the future, their cost and impact on the evolution of the

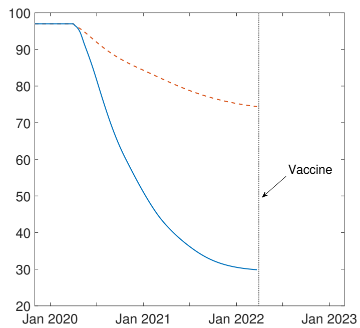
Figure 4: Constant Infectiousness



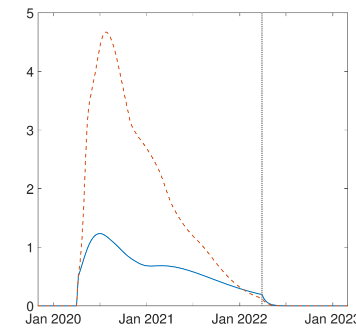
(a) Deceased



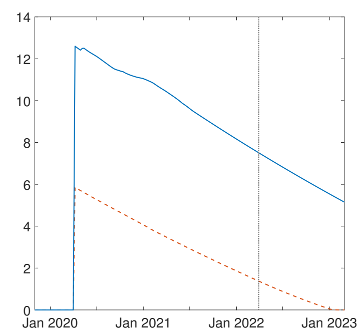
(b) Lockdowns



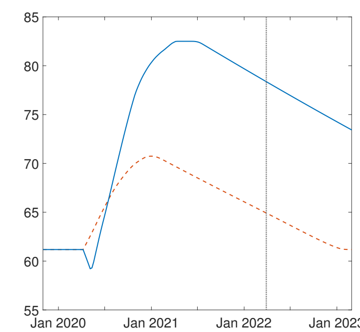
(c) Susceptible



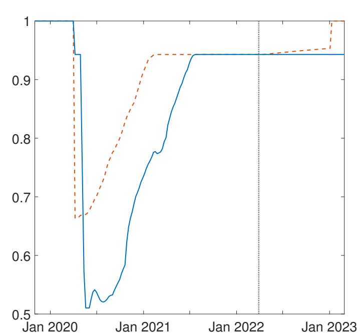
(d) Infected



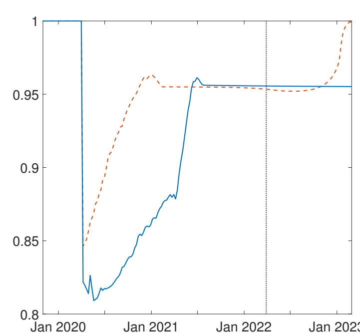
(e) Spread



(f) Debt



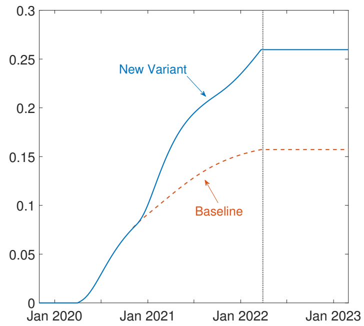
(g) Output



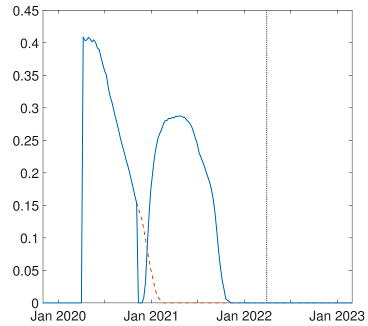
(h) Consumption

Notes: Deceased, Infected, and Susceptible are expressed in percentage of population. Debt is relative to pre-pandemic output. Spreads are in percentage relative to January 2020 value. Output and consumption are relative to pre-pandemic levels.

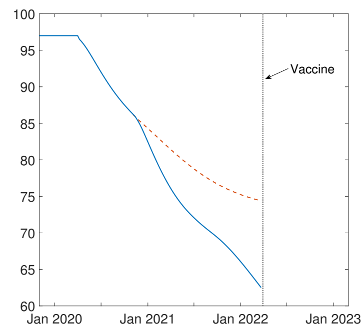
Figure 5: The Emergence of a New Variant



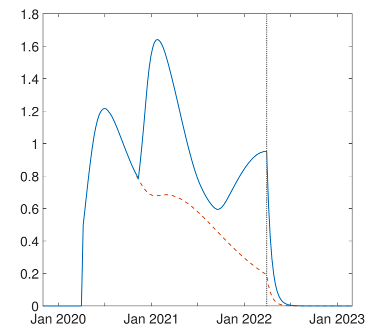
(a) Deceased



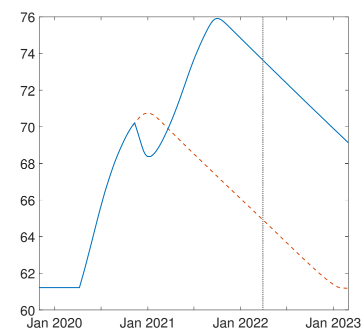
(b) Lockdowns



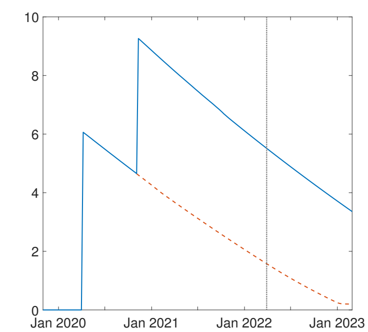
(c) Susceptible



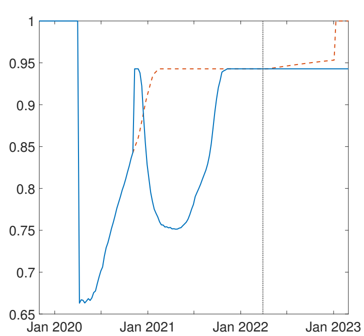
(d) Infected



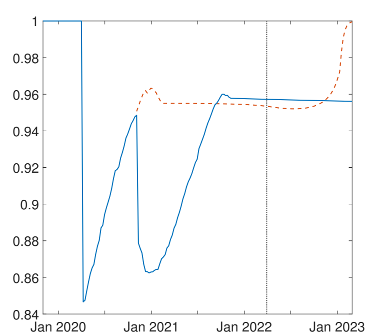
(e) Debt



(f) Spread



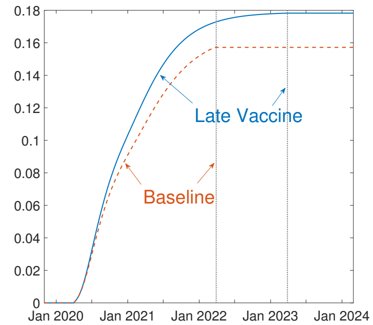
(g) Output



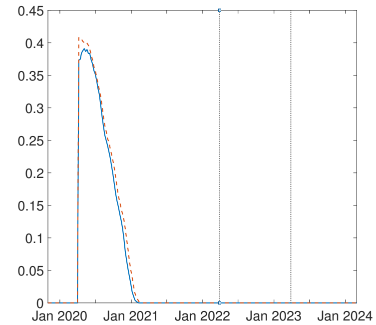
(h) Consumption

Notes: Deceased, Infected, and Susceptible are expressed in percentage of population. Debt is relative to pre-pandemic output. Spreads are in percentage relative to January 2020 value. Output and consumption are relative to pre-pandemic levels.

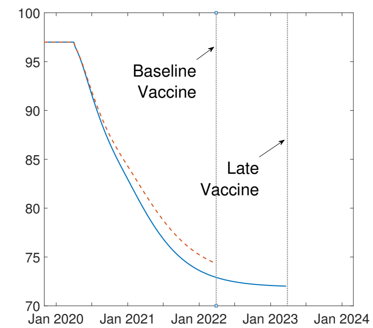
Figure 6: Later Vaccine Availability



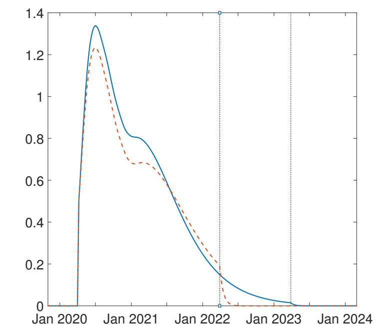
(a) Deceased



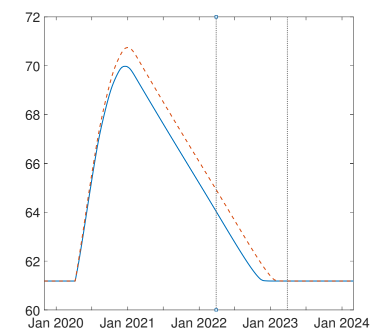
(b) Lockdowns



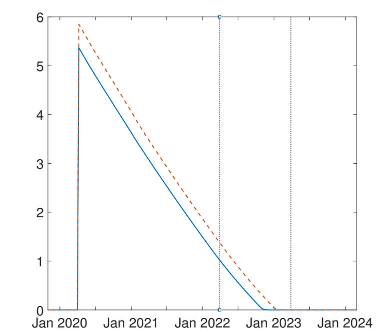
(c) Susceptible



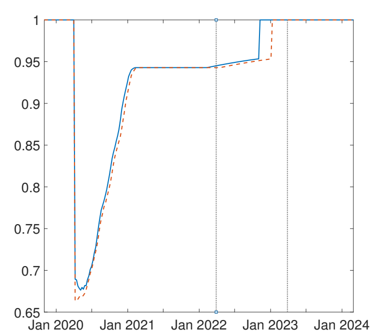
(d) Infected



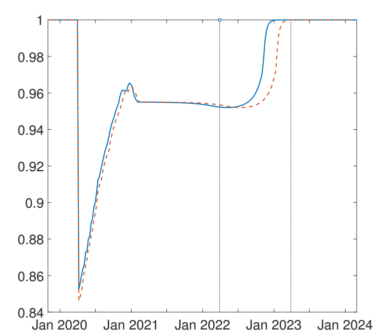
(e) Debt



(f) Spread



(g) Output



(h) Consumption

Notes: Deceased, Infected, and Susceptible are expressed in percentage of population. Debt is relative to pre-pandemic output. Spreads are in percentage relative to January 2020 value. Output and consumption are relative to pre-pandemic levels.

epidemic matter little. In Figure 6, we compare our baseline scenario with one in which the arrival of the vaccine is delayed by an additional year. In both scenarios, "herd immunity" is reached well in advance of the arrival of the highly effective vaccine that quickly ends the pandemic. Even so, because of the longer time horizon, the scenario with a vaccine in three years after the outbreak features an eventual death toll that is higher by about 0.02% of initial population. The increase arises because more time before the vaccine means more opportunities for infections and also because lockdowns are endogenously slightly less aggressive as can be seen in panel (b). The associated debt crisis is slightly less severe, as is apparent from panels (e) and (f). Panels (g) and (h) show that the impact of the time horizon to vaccination on output and consumption is also modest, except for small timing differences in late 2022. We conclude that the main findings of our analysis from the baseline scenario are qualitatively and quantitatively robust to the timing of the vaccine.

5 Conclusion

We have studied the COVID-19 epidemic in emerging markets by integrating epidemiological dynamics in a sovereign debt model and found that doing so can replicate salient features of the data on deaths, social distancing, and sovereign spreads. The pandemic is associated with many fatalities, deep recessions, and worsened fiscal conditions, including default. With financial frictions, the death toll and severity of the debt crisis reinforce each other as social distancing becomes increasingly costly.

By comparing our model with an otherwise equivalent one with perfect financial markets, we found that about a third of the welfare cost of the pandemic episode is due to default risk. Motivated by this finding, we asked whether debt relief programs that ease financial conditions could cushion the impact of the pandemic. We find that they deliver sizable social gains that accrue mainly to the country and to a lesser extent to its private lenders, with no cost to financial assistance entities orchestrating the programs. In light of these results, we find that the recent debt relief policies promoted by the International Monetary Fund and other international organizations are appropriate and timely for combating the costs associated with COVID-19. We hope that our work contributes to the discussion on the optimal domestic and international policy response to epidemic outbreaks in emerging markets.

Our results relied unavoidably on several abstractions that in turn open up promising avenues for subsequent work. We did not address the decentralization of optimal lockdowns in our model and the related question of whether the private sector alone would under- or over-provide social distancing. Note that in our setting, the private sector could depress activity excessively, as it may not internalize the effects on the fiscal conditions of the sovereign, including the resulting debt crisis. This stands in contrast with the better-understood externality whereby the private sector does not social distance enough, as agents do not internalize the role they might play in contagion and deaths in the broader economy. Finally, we conjecture that the mechanism we explored, relating financial conditions to mitigation efforts, could also operate at the individual level. For example, wealthier and financially unconstrained agents might lock down and isolate more aggressively, while agents unable to smooth consumption through borrowing might choose to expose themselves and others to increased risk. We further suspect that such a cross-sectional pattern would be salient not only in emerging markets but in advanced economies as well.

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Online Appendix to "Deadly Debt Crises: COVID-19 in Emerging Markets"

by Cristina Arellano, Yan Bai, and Gabriel Mihalache

A Recursive Formulation before the Epidemic

The recursive problem for the government before the epidemic resembles the one in Arellano, Mateos-Planas, and Ríos-Rull (2019). We assume that the government does not expect the epidemic to arise in the future, the probability of dying is expected to always be zero, and the measure of total population remains at one throughout. We study a Markov problem for the government. In period t, with debt holding B_t, the government chooses new issuance ℓ_t and partial default intensity $d_t \in [0,1]$ to solve

$$V^{\text{pre}}(B_t) = \max_{\ell_t,\, d_t \in [0,1]} u(c_t) + \beta V^{\text{pre}}(B_{t+1}), \tag{24}$$

subject to the evolution of the debt in equation (4), the resource constraint of the economy (3) with $N_t = 1$, and the bond price function $q^{\text{pre}}(B_{t+1})$. The Markov structure generates a time-invariant bond price schedule that depends on future default and borrowing decisions:

$$q^{\text{pre}}(B_{t+1}) = \frac{1}{1+r}\left\{(\delta + r)(1 - d_{t+1}(B_{t+1})) + [1 - \delta + \kappa(\delta + r)d_{t+1}(B_{t+1})]\, q^{\text{pre}}(B_{t+2}(B_{t+1}))\right\}. \tag{25}$$

This problem induces in pre-epidemic decision rules for the evolution of government debt $B_{t+1} = \mathbf{B}^{\text{pre}}(B_t)$, default $d_t = \mathbf{d}^{\text{pre}}(B_t)$, and per capita consumption $c_t = \mathbf{c}^{\text{pre}}(B_t)$. It also delivers the bond price schedule $q^{\text{pre}}(B_{t+1})$ and value function $V^{\text{pre}}(B_t)$. We use these results to set terminal conditions for the following problem during the epidemic. In the baseline experiment, we use the steady state values for debt B_t from this problem as its initial condition.

B Definition of Epidemic Equilibrium

The epidemic equilibrium consists of sequences of functions for consumption $\mathbf{c_t}(\mu_t, B_t)$, the government's borrowing policy $\mathbf{B_{t+1}}(\mu_t, B_t)$, default $\mathbf{d_t}(\mu_t, B_t)$, and lockdown $\mathbf{L_t}(\mu_t, B_t)$; the value function $V_t(\mu_t, B_t)$; the bond price schedule $q_t(\mu_t, B_{t+1})$; and the epidemiological laws of motion $\boldsymbol{\mu_{t+1}}(\mu_t, B_t)$ that summarizes the mass of susceptible, infected, and recovered for period $t = 0, 1, 2, ...$ such that, given the initial state (μ_0, B_0) and the availability of the vaccine in period H,

(i) For periods $t > H$, the epidemic is eliminated, $\mu_t^S = 0$, $\mu_t^I = 0$, and $\mu_t^R = \mu_H^R + \mu_H^S + (1 - \pi_D^0/\pi_I)\mu_H^I$ under the assumption that at period H, a fraction π_D^0/π_I of the infected dies and the rest recover. The optimal lockdown intensity is zero, $\mathbf{L_t}(\mu_t, B_t) = 0$. The government's borrowing and default policies, the value function, and the bond price schedule are the same as the pre-epidemic ones, $V_t(\mu_t, B_t) = V^{\text{pre}}(B_t)$, $\mathbf{d_t}(\mu_t, B_t) = \mathbf{d}^{\text{pre}}(B_t)$, $\mathbf{B_{t+1}}(\mu_t, B_t) = \mathbf{B}^{\text{pre}}(\mu_t, B_t)$, and $\mathbf{q_t}(\mu_t, B_{t+1}) = \mathbf{q}^{\text{pre}}(B_{t+1})$.

(ii) For any period $t \leq H$, taking as given the value function and the bond price schedule for period $t+1$, the government's value function and policies solve

$$V_t(\mu_t, B_t) = \max_{B_{t+1},\, d_t \in [0,1],\, L_t \in [0,1]} [u(c_t) - \chi \phi_{D,t}] + \beta V_{t+1}(\mu_{t+1}(\mu_t, L_t), B_{t+1}),$$

subject to the resource constraint

$$N_t c_t + (1 - d_t)(\delta + r) B_t = \tilde{z}\phi(d_t) N_t (1 - L_t) + q_t(\mu_{t+1}(\mu_t, L_t), B_{t+1})(B_{t+1} - (1-\delta)B_t),$$

the SIR laws of motion (6)-(9), deaths $\phi_{D,t} = \pi_D(\mu_t^I)\mu_t^I$ for $t < H$ and $\phi_{D,t} = \pi_D^0/\pi_I \mu_t^I$ for $t = H$, and $N_t = \mu_t^S + \mu_t^I + \mu_t^R$.

(iii) For any period $t \leq H$, taking as given the government's policies in period $t+1$ and the epidemiological state, the bond price schedule satisfies

$$q_t(\mu_{t+1}(\mu_t, L_t), B_{t+1}) = \frac{1}{1+r}\left\{(\delta + r)(1 - \mathbf{d_{t+1}}) + [1 - \delta + \kappa(\delta + r)\mathbf{d_{t+1}}]\, q_{t+1}(\boldsymbol{\mu_{t+2}}, \mathbf{B_{t+2}})\right\}.$$

(iv) The evolution of the epidemiological state $\boldsymbol{\mu_{t+1}}(\mu_t, B_t)$ is consistent with the SIR laws of motion (6)-(9) and equilibrium social distancing $L_t = \mathbf{L_t}(\mu_t, B_t)$.

C Reference Models

Perfect financial markets. In period 0, the government chooses sequences of consumption $\{c_t\}$ and lockdowns $\{L_t\}$, with $L_t \in [0,1]$, to maximize its lifetime value (1), subject to the lifetime budget constraint

$$\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} N_t c_t = \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} \tilde{z}[N_t(1 - L_t)]^\alpha, \tag{26}$$

the SIR laws of motion (6)-(9) and the total population constraint (11). For an initial debt level B_0, we back out long-term borrowing, with the same maturity structure as in the baseline, recursively from the per-period budget constraints, using the optimal allocations, lockdowns, and SIR dynamics, for any t:

$$q^f B_{t+1} = N_t c_t + [(\delta + r) + q^f(1-\delta)]B_t + \tilde{z}[N_t(1 - L_t)]^\alpha,$$

where the risk-free, long-term bond price q^f satisfies

$$q^f = \frac{1}{1+r}[(\delta + r) + (1-\delta)q^f].$$

Because of our choice of normalization for the payments scheduled by each unit of debt, we have $q^f = 1$. As pre-pandemic steady state output in weeks is 1, we can interpret B_t not only as abstract bond units but also as multiple of weekly output; thus, a B_t of 52 would imply a debt-to-output ratio of 100%.

Financial autarky. In this model, the government cannot borrow or lend internationally. It chooses consumption and lockdown to maximize its lifetime value (1), subject to period-by-period budget constraints,

$$N_t c_t = \tilde{z}[N_t(1 - L_t)]^{\alpha}; \tag{27}$$

the SIR lows of motion (6)-(9), and the total population constraint (11).

D Proofs

Before proceeding to the proofs of propositions 1 and 2, we first set up the stripped-down, two-period version of the model from Section 3. The economy starts with zero debt $B_0 = 0$ and population N_0, which consists of measure μ_0^S susceptible, μ_0^I infected, and μ_0^R recovered individuals; that is, $N_0 = \mu_0^S + \mu_0^I + \mu_0^R$. In the last period, period 1, the government cannot issue more debt, $B_2 = 0$.

The government has limited commitment and can choose to default on its debt. With zero initial debt, the government will not default in period 0. It chooses consumption (c_0, c_1) and lockdowns (L_0, L_1) in both periods and borrowing B_1 and partial default $d_1 \in [0, 1]$ in period 1 to maximize its objective

$$\max\left[u(c_0) - \chi\phi_{D,0}\right] + \beta\left[u(c_1) - \chi\phi_{D,1}(L_0)\right], \tag{28}$$

subject to the budget constraints of each period,

$$\begin{aligned} N_0 c_0 &= \tilde{z} N_0(1 - L_0) + q_0(B_1)B_1, \\ N_1 c_1 + (1 - d_1)B_1 &= \tilde{z}\gamma(d_1)N_1(1 - L_1); \end{aligned}$$

the evolution of infected, susceptible, and recovered

$$\mu_1^I(L_0) = \pi_I \mu_0^I + \pi_{SI}(1 - \theta L_0)^2 \mu_0^I \mu_0^S, \tag{29}$$

$$\mu_1^S(L_0) = \mu_0^S - \pi_{SI}(1 - \theta L_0)^2 \mu_0^I \mu_0^S, \tag{30}$$

$$\mu_1^R = \mu_0^R + (1 - \pi_I)\mu_0^I - \phi_{D,0}; \tag{31}$$

the fatalities induced by infection

$$\phi_{D,1}(L_0) = \pi_D(\mu_1^I(L_0))\mu_1^I(L_0); \tag{32}$$

the evolution of total population

$$N_1 = N_0 - \phi_{D,0}; \tag{33}$$

and the bond price schedule

$$q_0(B_1) = (1 - d_1(B_1))/(1 + r). \tag{34}$$

It is easy to see that the government will not impose any lockdown in the last period, $L_1 = 0$, since doing so does not affect the spread of the disease and death. The government also cannot influence the death toll in period 0, $\phi_{D,0}$, nor can it influence the total population in period 1, N_1. Given that both $\phi_{D,0}$ and N_1 are determined by the initial level of infection and epidemiology parameters, we assume for simplicity that $\phi_{D,0} = 0$.

In the stripped-down version of the model, we make the following assumptions to guarantee interior solutions to partial default $d_1 \in [0,1]$ and lockdown $L_0 \in [0,1]$ to better illustrate the interaction mechanism between the health and debt crisis. These assumptions are relaxed in the general model.

Assumptions

1. The discount factor is such that $\beta(1+r) \leq 1$. The lockdown effectiveness takes the value of 1, $\theta = 1$. The case fatality rate is constant, $\pi_D(\mu_0^I) = \pi_D^0$. The value of life χ satisfies

$$\chi \geq \frac{\tilde{z}^{1-\sigma}}{2\beta\pi_D^0\pi_{SI}\mu_0^I\mu_0^S}\left(1+\frac{1}{1+r}\right)^{-\sigma}\lambda^{-\sigma},$$

where $\lambda = \left[1+\beta^{\frac{1}{\sigma}}(1+r)^{\frac{1-\sigma}{\sigma}}\right]^{-1} < 1$.

2. The function $\gamma : [0,1] \to \mathbb{R}$ is differentiable, decreasing $\gamma' < 0$, concave $\gamma'' < 0$, and with $\gamma(0) = 1$ and $\gamma'(0) = 0$.

3. Let $\bar{d}$ be the maxima of the function $f(d) = -(1-d)\gamma'(d)$. We assume $f(\bar{d}) \geq \lambda$.

Note that Assumption 2 implies $\gamma'(d)$, and so $f(d)$ are continuous functions. By the maximum theorem, there exists a $\bar{d} \in [0,1]$ that maximizes the f function. Furthermore, $f(0) = f(1) = 0$, since $\gamma'(0) = 0$. It has to be the case that $0 < \bar{d} < 1$.[19]

D.1 Proof of Proposition 1

Proof. We start with the baseline's equilibrium conditions under the assumption of interior solutions for default and lockdown. We then verify that Assumptions 1-3 guarantee that the solution is indeed interior. The equilibrium default, borrowing, consumption, and lockdown, $\{d_1, B_1, c_1, c_0, L_0\}$, satisfy the following five equations:

$$-\tilde{z}\gamma'(d_1)N_1 = B_1, \tag{35}$$

$$u'(c_0) = \beta(1+r_d(B_1))u'(c_1), \tag{36}$$

$$N_1c_1 + (1-d_1)B_1 = \tilde{z}\gamma(d_1)N_1, \tag{37}$$

$$c_0 = \frac{1}{1+\frac{1}{1+r}[\beta(1+r_d(B_1))]^{1/\sigma}}\left(\tilde{z}(1-L_0) + \frac{1}{1+r}\tilde{z}\gamma(d_1)\right), \tag{38}$$

$$\tilde{z}u'(c_0) = \beta\chi\left(-\frac{\partial\phi_{D,1}(L_0)}{\partial L_0}\right), \tag{39}$$

where $1+r_d(B_1) = (1+r)/(1-\eta(B_1))$ is the domestic interest rate, with $\eta(B_1)$ defined as the elasticity of bond price with respect to borrowing $\eta(B_1) = -\partial\ln(q_0)/\partial\ln(B_1) \geq 0$. Equation (35) is the first order condition on partial default d_1 in period 1; equation (36) is the Euler equation for borrowing B_1; equation

19. An example of γ function satisfying assumptions 2 and 3 is $\gamma(d) = 1 - \gamma_0 d^{\gamma_1}$ with $\gamma_1 > 1$ and $\gamma_0 \geq \lambda((\gamma_1-1)/\gamma_1)^{1-\gamma_1}$. In this case, $\gamma'(d) < 0$, $\gamma''(d) < 0$, $\gamma(0) = 1$, and $\gamma'(0) = 0$. The maxima is $\bar{d} = (\gamma_1-1)/\gamma_1$ with $0 < \bar{d} < 1$. At the maxima, $f(\bar{d}) \leq \lambda$, since $\gamma_0 \geq \lambda((\gamma_1-1)/\gamma_1)^{1-\gamma_1}$.

(37) is period 1's budget constraint; equation (38) results from the sum of budget constraints in both periods and the Euler equation (36); and equation (39) comes from the first order condition for lockdown.

Using equation (35)-(38), we can show that d_1 satisfies the following equation for any lockdown level L_0:

$$M(d_1; L_0) \equiv -(1-d_1)\tilde{z}\gamma'(d_1) - \left\{ \frac{\tilde{z}\gamma(d_1)}{1+\frac{1}{1+r}[\beta(1+\tilde{r}_d(d_1))]^{1/\sigma}} - \frac{[\beta(1+\tilde{r}_d(d_1))]^{1/\sigma}\tilde{z}(1-L_0)}{1+\frac{1}{1+r}[\beta(1+\tilde{r}_d(d_1))]^{1/\sigma}} \right\} = 0, \quad (40)$$

where $\tilde{r}_d(d_1)$ is the domestic interest rate after incorporating the optimal condition of d_1 (35) and the bond price schedule (34); that is,

$$\tilde{r}_d(d_1) = \frac{1+r}{1-\frac{\gamma'(d_1)}{\gamma''(d_1)(1-d_1)}} \geq 1+r.$$

Let the solution to equation (40) be $d_1 = H_d(L_0)$. We first verify that for any L_0, partial default is interior; that is, $0 < d_1 < 1$. When $d_1 = 0$,

$$M(0; L_0) = -\tilde{z} \left\{ \frac{1-[\beta(1+r)]^{1/\sigma}(1-L_0)}{1+\frac{1}{1+r}[\beta(1+r)]^{1/\sigma}} \right\} < 0,$$

which holds as $\gamma'(0) = 0$, $\gamma(0) = 1$ and $\beta(1+r) < 1$ by assumptions 2 and 1. When $d_1 = \bar{d}$,

$$\begin{aligned} M(\bar{d}; L_0) &= \tilde{z}f(\bar{d}) - \left\{ \frac{\tilde{z}\gamma(\bar{d})}{1+\frac{1}{1+r}[\beta(1+r_d(\bar{d}))]^{1/\sigma}} - \frac{[\beta(1+r_d(\bar{d}))]^{1/\sigma}\tilde{z}(1-L_0)}{1+\frac{1}{1+r}[\beta(1+r_d(\bar{d}))]^{1/\sigma}} \right\} \\ &\geq \tilde{z}f(\bar{d}) - \tilde{z} \left\{ \frac{1}{1+\frac{1}{1+r}[\beta(1+r)]^{1/\sigma}} \right\} = \tilde{z}[f(\bar{d}) - \lambda] \geq 0. \end{aligned}$$

The first inequality holds because $\gamma(\bar{d}) \leq \gamma(0) = 1$, $r_d(\bar{d}) \geq r$, and $[\beta(1+r_d(\bar{d}))]^{1/\sigma}\tilde{z}(1-L_0) \geq 0$. The last inequality holds because of Assumption 3. By the intermediate value theorem, for any L_0, there exists an interior partial default $0 < d_1 \leq \bar{d} < 1$ such that $M(d_1; L_0) = 0$.

The equilibrium without an epidemic satisfies equations (35)-(38) with $L_0 = 0$, since there is no benefit of locking down without an outbreak. Namely, the equilibrium consumption, borrowing and default with epidemic are the same as those without an epidemic if the optimal lockdown level is zero. Hence, we need to prove that for any optimal lockdown level $L_0 > 0$, the corresponding d_1 is also higher; that is, $\partial H_d(L_0)/\partial L_0 \geq 0$.

We now show the partial default is higher with epidemic:

$$\frac{\partial H_d}{\partial L_0} = -\frac{\partial M/\partial L_0}{\partial M/\partial d_1} \geq 0.$$

The inequality holds because $\partial M/\partial L_0 \leq 0$ and $\partial M/\partial d_1 \geq 0$ for $d_1 \leq \bar{d}$. Hence, the optimal default with an epidemic is higher than default without one, owing to positive lockdown. □

D.2 Proof of Proposition 2

Proof. Under perfect financial markets, the government commits to repay its debt and chooses consumption (c_0, c_1) and lockdown (L_0, L_1) to maximize its objective (28), subject to the lifetime budget constraint

$$N_0 c_0 + \frac{1}{1+r} N_1 c_1 = \tilde{z} N_0 (1 - L_0) + \frac{1}{1+r} \tilde{z} N_1 (1 - L_1),$$

fatalities (32), SIR laws of motion (29)-(31), and the evolution of population (33). Recall that we assume $\phi_{D,0} = 0$, and so $N_1 = N_0$. In this case, we can back up the optimal borrowing from period 0's budget constraint $B_1 = (1+r)[N_0 c_0 - \tilde{z} N_0 (1 - L_0)]$. Let the optimal consumption and lockdown under perfect financial markets be (c_0^e, c_1^e, L_0^e). Similarly, as in (38) in the baseline, consumption c_0 is a share of the lifetime income:

$$c_0^e = \frac{1}{1 + \frac{1}{1+r}[\beta(1+r)]^{1/\sigma}} \left(\tilde{z}(1 - L_0^e) + \frac{1}{1+r} \tilde{z} \right).$$

The optimal lockdown L_0^e weights the marginal benefit and cost as in (39). Plugging in the optimal consumption c_0^e into (39), we find the equation characterizing optimal lockdown under perfect financial markets:

$$\left(\frac{\tilde{z}(1 - L_0) + \frac{1}{1+r} \tilde{z}}{1 + \frac{1}{1+r}[\beta(1+r)]^{1/\sigma}} \right)^{-\sigma} \tilde{z} = 2\beta\chi \pi_D^0 \pi_{SI} \mu_0^I \mu_0^S (1 - L_0), \tag{41}$$

where the right-hand side is the marginal benefit of lockdown—that is, $\beta\chi \left(-\frac{\partial \phi_{D,1}(L_0)}{\partial L_0} \right)$. According to Assumption 1, the left-hand side is lower than the right-hand side when $L_0 = 0$, and the left-hand side is higher than the right-hand side when $L_0 = 1$. Hence, by the intermediate value theorem, the optimal L_0^e is between 0 and 1.

Similarly, plugging in the consumption (38) and default intensity (35) into the first order condition for lockdown (39), we can show that the optimal lockdown in our baseline satisfies

$$\left(\frac{\tilde{z}(1 - L_0) + \frac{1}{1+r} \tilde{z} \gamma(d_1)}{1 + \frac{1}{1+r}[\beta(1+\tilde{r}_d(d_1))]^{1/\sigma}} \right)^{-\sigma} \tilde{z} = 2\beta\chi \pi_D^0 \pi_{SI} \mu_0^I \mu_0^S (1 - L_0). \tag{42}$$

As in the perfect financial markets case, there is an interior solution for lockdown. It is easy to show that under $d_1 = 0$, the optimal lockdown in our baseline is the same as the one from perfect financial markets L_0^e. However, with $d_1 > 0$, for the same level of L_0, the left-hand side of (42) becomes larger than that of (41) owing to both lower $\gamma(d_1)$ and a higher domestic interest rate $\tilde{r}_d(d_1)$ in the baseline model. Hence the marginal benefit of locking down is smaller as long as $d_1 > 0$, which leads to a lower lockdown level than L_0^e. In summary, the lockdown level in our baseline is equal to or lower than L_0^e. With lower lockdown, the death toll $\phi_{D,1}$ is higher in our baseline model. □

E Computational Algorithm

In this appendix, we describe the computation of our model. We augment the original problem with taste shocks on debt choices B', following Dvorkin, Sanchez, Sapriza, and Yurdagul (2018) and Gordon

(2019), and then we describe the computation algorithm, which solves backwards in time for value and policy functions.

The taste shock on B' helps numerical stability and the convergence properties of our model with long-term, defaultable debt. The set of B is discrete, and each element in the set is associated with an iid taste shock, distributed Gumbel (Extreme Value Type I). The parameter controlling the magnitude of taste shocks is ρ_B.

Model with taste shocks. For the purposes of this appendix, we switch the state from $(\mu_t^S, \mu_t^I, \mu_t^R)$, the one in the main text, to $(\mu_t^S, \mu_t^I, \mu_t^D)$. Given that the sum of the four group is always 1, $\mu_t^S + \mu_t^I + \mu_t^R + \mu_t^D = 1$, the two formulations are equivalent. With slight abuse of notation, we use $\mu_t = (\mu_t^S, \mu_t^I, \mu_t^D)$ as the SIR state variable. The government's problem becomes

$$W_t\left(\mu_t, B_t, B_{t+1}\right) = \max_{L_t, d_t} \; u(c_t) - \pi_D(\mu_t^I)\mu_t^I + \beta V_{t+1}\left(\mu_{t+1}(\mu_t, L_t), B_{t+1}\right), \tag{43}$$

subject to the resource constraint and the SIR laws of motion

$$c_t = \tilde{z}\phi(d_t)(1 - L_t) + \frac{\left\{B_{t+1} - \left[1 - \delta + \kappa(\delta + r)d_t\right] B_t\right\} q_t(\mu_{t+1}(\mu_t, L_t), B_{t+1}) - (\delta + r)(1 - d_t)B_t}{1 - \mu_t^D} \tag{44}$$

$$\mu_t^x = \pi_{SI}(1 - \theta L_t)^2 \mu_t^S \mu_t^I \tag{45}$$
$$\mu_{t+1}^S = \mu_t^S - \mu_t^x \tag{46}$$
$$\mu_{t+1}^I = \pi_I \mu_t^I + \mu_t^x \tag{47}$$
$$\mu_{t+1}^D = \mu_t^D + \pi_D(\mu_t^I)\mu_t^I. \tag{48}$$

Let the optimal default and lockdown choices be $d_t(\mu_t, B_t, B_{t+1})$ and $L_t(\mu_t, B_t, B_{t+1})$, respectively. The choice probabilities over B_{t+1} are given by

$$Pr\left(B_{t+1} | \mu_t, B_t\right) = \frac{\exp((W_t(\mu_t, B_t, B_{t+1}) - \overline{W}_t(\mu_t, B_t))/\rho_B)}{\sum_{\tilde{B}_{t+1}} \exp((W_t(\mu_t, B_t, \tilde{B}_{t+1}) - \overline{W}_t(\mu_t, B_t))/\rho_B)},$$

where $\overline{W}$ is the maximum option net of taste shocks, $\overline{W}_t(\mu_t, B_t) = \max_{B_{t+1}} W_t(\mu_t, B_t, B_{t+1})$. The value V, in expectation over taste shocks, satisfies

$$V_t(\mu_t, B_t) = \overline{W}_t(\mu_t, B_t) + \rho_B \log\left\{\sum_{B_{t+1}} \exp\frac{W_t(\mu_t, B_t, B_{t+1}) - \overline{W}_t(\mu_t, B_t)}{\rho_B}\right\}.$$

We write the bond price schedule for any choices (L_t, B_{t+1}) in state μ_t as

$$\begin{aligned} q_t\left(\mu_{t+1}, B_{t+1}\right) = \frac{1}{1+r} \sum_{B_{t+2}} \Pr\left(B_{t+2} | \mu_{t+1}, B_{t+1}\right) \times \\ \left\{(\delta + r)(1 - d_{t+1}^*) + \left[1 - \delta + \kappa(\delta + r)d_{t+1}^*\right] q_{t+1}\left(\mu_{t+2}(\mu_{t+1}, L_{t+1}^*), B_{t+2}\right)\right\}, \end{aligned}$$

where $d^*_{t+1} = d_{t+1}(\mu_{t+1}, B_{t+1}, B_{t+2})$ and $L^*_{t+1} = L_{t+1}(\mu_{t+1}, B_{t+1}, B_{t+2})$ and equilibrium policies evaluate at the relevant future state.

Computational algorithm. Note that in period t, the state variable μ^D_t affects the period t allocation only through the per-capita debt burden $B_t/(1-\mu^D_t)$ and per-capita borrowing $B_{t+1}/(1-\mu^D_t)$. In particular, μ^D_t does not affect current or future losses from death. At our benchmark SIR parameters, the eventual death toll is less than 1% without lockdowns, and hence its impact on per capita debt is small. Therefore, we approximate the original problem with a simplified one with SIR state variables restricted to (μ^S_t, μ^I_t) and rewrite the resource constraint as

$$c_t = \tilde{z}\phi(d_t)(1-L_t)^\alpha + \{B_{t+1} - [1-\delta+\kappa(\delta+r)d_t]\, B_t\}\, q_t(\mu_{t+1}(\mu_t, L_t), B_{t+1}) - (\delta+r)(1-d_t)B_t. \quad (49)$$

We verified that the simplified problem approximates well the original one. This simplification, however, reduces the state space by one variable and dramatically saves computation time.

We first solve the stationary equilibrium, which both is the solution for the pre-epidemic equilibrium and governs behavior after the introduction of the vaccine. We then solve backwards over time the equilibrium under SIR dynamics.

1. Stationary equilibrium

   (a) Guess value function $V^s(B)$ and bond price schedule $q^s(B')$.

   (b) For each (B, B'), solve for the optimal default decision. We find the solution d^* of the following first order condition for partial default,

   $$-\tilde{z}\phi'(d) = (\delta+r)B\left[1-\kappa q^s(B')\right].$$

   Evaluate the consumption per capita c from the resource constraint

   $$c = \tilde{z}\phi(d) + \left\{B' - [1-\delta+\kappa(\delta+r)d]\, B\right\} q^s(B') - (\delta+r)(1-d)B$$

   at three default choice levels, $d = \{d^*, 0, 1\}$, and pick the default intensity that results in the highest consumption per capita under (B, B'). Let the optimal default be $d^s(B, B')$ and the corresponding consumption be $c^s(B, B')$.

   (c) Compute the value from (B, B') and the optimal default intensity $d^s(B, B')$. Then, we evaluate $W^s(B, B') = u(c^s(B, B')) + \beta V^s(B')$.

   (d) Calculate the probability of each B',

   $$\Pr\left(B'|B\right) = \frac{\exp((W^s(B,B') - \overline{W}^s(B))/\rho_B)}{\sum_{\tilde{B}'}\exp((W^s(B,\tilde{B}') - \overline{W}^s(B))/\rho_B)},$$

   and the maximum W^s for each b, $\overline{W}^s(B) = \max_{B'} W^s(B, B')$.

   (e) Update $V^s(B)$, $V^s(B) = \overline{W}^s(B) + \rho_B \log\left\{\sum_{B'} \exp \frac{W^s(B,B') - \overline{W}^s(B)}{\rho_B}\right\}$.

(f) Update the bond price schedule $q^s(B')$,

$$q^s(B') = \frac{1}{1+r}\sum_{B''}\Pr(B''|B')\left\{(\delta+r)(1-d(B',B'')) + \left[1-\delta+\kappa(\delta+r)d^s(B',B'')\right]q^s(B'')\right\}.$$

(g) Check whether V^s and q^s converged. If both did, we are done. Otherwise, go back to step 1(b).

2. Period H problem (with vaccine)
In period H, a vaccine is available. All susceptible individuals are marked as recovered; a fraction π_D^0/π_I of the infected dies, while the rest recover:

$$W_H(\mu_H, B_H, B_{H+1}) = \max_{d_H}\left\{u(c_H) - \left[\frac{\pi_D^0}{\pi_I}\right]\mu_H^I\chi + \beta V^s(B_{H+1})\right\}, \tag{50}$$

subject to the resource constraint (49). Let the solution be $V_H^{\{0\}}(\mu_H, B_H)$, $d_H^{\{0\}}(\mu_H, B_H, B_{H+1})$, $Pr^{\{0\}}(B_{H+1}|\mu_H, B_H)$, $q_H^{\{0\}}(\mu_{H+1}, B_{H+1})$, and $L_H^{\{0\}}(\mu_H, B_H, B_{H+1}) = 0$.

3. Period $t < H$ problem

(a) Start with $q_{t+1}^{\{0\}}(\mu_{t+2}, B_{t+2})$, $d_{t+1}^0(\mu_{t+1}, B_{t+1}, B_{t+2})$, $L_{t+1}^0(\mu_{t+1}, B_{t+1}, B_{t+2})$, $Pr^{\{0\}}(B_{t+2}|\mu_{t+1}, B_{t+1})$, and $V_{t+1}^{\{0\}}(\mu_{t+1}, B_{t+1})$.

(b) Construct bond price $q_t^{\{1\}}(\mu_{t+1}, B_{t+1})$:

$$q_t^{\{1\}}(\mu_{t+1}, B_{t+1}) = \frac{1}{1+r}\sum_{B_{t+2}}\Pr^{\{0\}}(B_{t+2}|\mu_{t+1}, B_{t+1})\Big\{(\delta+r)(1-d_{t+1}^{\{0\}}) + \left[1-\delta+\kappa(\delta+r)d_{t+1}^{\{0\}}\right]q_{t+1}^{\{0\}}\left(\mu_{t+2}(\mu_{t+1}, L_{t+1}^{\{0\}}), B_{t+2}\right)\Big\},$$

where $d_{t+1}^{\{0\}} = d_{t+1}^{\{0\}}(\mu_{t+1}, B_{t+1}, B_{t+2})$ and $L_{t+1}^{\{0\}} = L_{t+1}^{\{0\}}(\mu_{t+1}, B_{t+1}, B_{t+2})$.

(c) Solve for the optimal default and lockdown policies for each B_t and B_{t+1} paid,

$$W_t(\mu_t, B_t, B_{t+1}) = \max_{L_t, d_t} u(c_t) - \pi_D(\mu_t^I)\mu_t^I\chi + \beta V_{t+1}^{\{0\}}(\mu_{t+1}(\mu_t, L_t), B_{t+1}),$$

subject to the resource constraint (49) and the SIR laws of motion. Specifically, we search over the grid of L_t. For each $(\mu_t, B_t, B_{t+1}, L_t)$, we find the solution d^* to the following equation:

$$-\tilde{z}\phi'(d)(1-L_t)^\alpha(1-\mu_t^D) = (\delta+r)B_t\left[1-\kappa q_t^{\{1\}}(\mu_{t+1}(\mu_t, L_t), B_{t+1})\right].$$

We pick the default choice in $\{d^*, 0, 1\}$ which yields the highest consumption per capita for $(\mu_t, B_t, B_{t+1}, L_t)$. Let the optimal default and lockdown choices be $d_t^{\{1\}}(\mu_t, B_t, B_{t+1})$ and $L_t^{\{1\}}(\mu_t, B_t, B_{t+1})$.

(d) Calculate the probability of choosing each B_{t+1}:

$$\Pr^{\{1\}}(B_{t+1}|\mu_t, B_t) = \frac{\exp((W_t(\mu_t, B_t, B_{t+1}) - \overline{W}_t(\mu_t, B_t))/\rho_B)}{\sum_{\tilde{B}_{t+1}}\exp((W_t(\mu_t, B_t, \tilde{B}_{t+1}) - \overline{W}_t(\mu_t, B_t))/\rho_B)},$$

with the maximum value given by $\overline{W}_t(\mu_t, B_t) = \max_{B_{t+1}} W_t(\mu_t, B_t, B_{t+1})$.

(e) Calculate the period t's value and bond price functions, for use at $t-1$:

$$V_t^{\{1\}}(\mu_t, B_t) = \overline{W}_t(\mu_t, B_t) + \rho_B \log \left\{ \sum_{B_{t+1}} \exp \frac{W_t(\mu_t, B_t, B_{t+1}) - \overline{W}_t(\mu_t, B_t)}{\rho_B} \right\}.$$

(f) Assign the functions with superscript $\{1\}$ to functions with superscript $\{0\}$. Go back to step 3(a) for the previous t until $t = 0$, then stop.