Micro Risks and Pareto Improving Policies[*]

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Abstract

We provide sufficient conditions for the feasibility of robust Pareto-improving (RPI) fiscal policies when the risk-free interest rate on government bonds is below the growth rate ($r < g$) or there is a markup between price and marginal cost. We do so in the class of incomplete markets models pioneered by Bewley-Huggett-Aiyagari, but we allow for an arbitrary amount of ex ante heterogeneity in terms of preferences and income risk. An RPI improve risk sharing and potentially guide the economy to a more efficient level of capital. We show that the elasticities of aggregate savings to changes in interest rates is a crucial ingredient that determines the feasibility of RPIs. We establish that debt and investment associated with an RPI may be complements along the transition, rather than the traditional substitutes.

1 Introduction

In this paper, we provide sufficient conditions for a Pareto improvement when the risk-free interest rate r on government bonds is below the growth rate ($r < g$) or the economy is subject to markups. We do so in the class of incomplete markets models pioneered by Bewley-Huggett-Aiyagari, but we allow for an arbitrary amount of ex ante heterogeneity in terms of preferences and income risk. We restrict fiscal policy to linear taxes or subsidies and non-negative lump-sum transfers as well as government debt.

The first step of our analysis is to define a new welfare metric, what we term "Robust Pareto Improvements" (RPI). Relative to an initial equilibrium, an RPI weakly increases every household's budget set at *every* idiosyncratic state and time. Specifically, all after-tax factor prices, as well as pure profits, if there are any, weakly increase at every date, with at least one factor price strictly increasing at some date. Moreover, lump sum transfers weakly increase, as well.[1] By weakly expanding the budget set of all agents at all dates, these policies necessarily generate a Pareto improvement, but do not require detailed knowledge on preferences or idiosyncratic risk, hence the term "robust." Note that this welfare criterion rules out tax and transfer schemes that trade off consumption in one date or state against another. This includes using the tax system to directly provide insurance or using lump sum taxation to relax the borrowing constraint.[2]

The second step of our analysis establishes when a feasible RPI exists. That is, when is it possible for the government, given its limited fiscal tools, to weakly increase all after-tax factor prices. We show that the feasibility of an RPI involves only knowledge of the initial aggregate state of the economy; the aggregate savings schedule – that is, the total private savings as a function of interest rates and government transfers; and the aggregate production function.

To understand the role of the aggregate savings schedule, we first study an economy without productivity or population growth such that $r < 0$ in the initial stationary equilibrium. It is natural to conjecture that a policy that increases government debt by some strictly positive amount could be helpful, as the interest rate is low. Issuing government bonds, however, may lead to an increase in interest rates that crowds out capital. Simply issuing debt, therefore, may eventually reduce output, wages and profits, which hurt households that rely on these sources of income and is not an RPI. The government, however, has additional policy instruments that could potentially be used to offset these price declines.

In particular, the government can provide a subsidy on the rental rate of capital that ensures capital remains unchanged, despite the increase in the interest rate on government bonds. This *constant-K* policy guarantees that capital, output, wages, and profits are all the same as in the initial equilibrium. If the government can finance the capital subsidy with just the revenue it receives from bond issuances, and lump-sum transfer any additional surplus, then this policy makes *every* household weakly better off: the return to wealth has increased, after-tax wages and profits have remained constant, and the government is providing a weakly positive lump-

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sum transfer at all dates.[3] Agents with positive assets will be strictly better off, and the policy generates a Pareto improvement.

Focusing on the steady state, we derive a simple necessary condition for an RPI to be feasible with a constant-K. Let B_{ss} denote the outstanding government debt at the steady state, r^o and r' denote the original and the new interest rates paid on debt to households, respectively, with $r^o < r' < 0$, and let K^o denote the initial capital stock. For the RPI to be feasible we require that[4]

$$-r'B_{ss} \geq (r' - r^o)K^o$$

The left-hand side is the revenue generated by the government issuance of bonds in steady state, as $r' < 0$. The right-hand side represents the fiscal cost of the subsidy to capital: the increase in the interest rate, $r' - r^o$, is the subsidy rate required to keep K constant, and K^o is the tax base. The left-hand side captures the level of debt the government is asking households to absorb, while the right-hand side reflects the increase in interest rates necessary to implement it in equilibrium. The key consideration is therefore whether households are willing to increase wealth without a large increase in interest rates—that is, the elasticity of the aggregate demand for savings is sufficiently large.

Interestingly, this potential Pareto improvement is achieved without increases in aggregate consumption or output at any date, as capital and labor remain at their initial levels, yet every household sees its budget set weakly expand at every date and idiosyncratic state, and hence every household perceives that it could increase consumption. In equilibrium, however, the higher interest rate induces some (high-income) households to postpone consumption, allowing others (low-income) to increase theirs, improving risk sharing, despite the absence of a progressive tax and transfer scheme. The aggregate saving elasticity being "large enough" is exactly when, in the *aggregate*, households balance the increased desire to save due to the higher interest rate against the increased desire to spend due to the expanded budget set, keeping aggregate consumption from increasing. The willingness to hold government debt rather than consume (in aggregate), despite feeling richer, echoes the result of Samuelson (1958), in which the "social contrivance" of money achieved a better allocation of a fixed endowment.

We extend this insight to the case of general policies, including those that potentially involve changes in capital. We study two cases.

First, suppose the economy has "over accumulated" capital such that the marginal product of capital (MPK) is less than the rate of depreciation. That is, capital is above the "Golden Rule" level, which implies that reductions in capital increase resources for consumption. In this case,

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[3]Contrast this with the utilitarian metric of Dávila, Hong, Krusell and Ríos-Rull (2012), which requires that a change in *relative* factor prices improve the lot of the poorest households relative to that of the richest.

[4]Here, we assume initial debt is zero. We relax this in the text.

under some weak regularity conditions, an RPI always exists. The intuition is similar to the canonical analysis of Diamond (1965), given that reducing capital increases resources available for consumption. The heterogeneous agent environment and the stricter RPI metric involves some additional work, but the result intuitively holds in the extended model.

Second, we consider the case when the MPK is greater than the rate of depreciation, which is likely the more realistic scenario. Given a markup, this can be consistent with a negative risk-free interest rate on bonds.[5] For small perturbations around the initial equilibrium, we show that the discounted sum of each t's aggregate saving elasticity to an interest rate change at some date τ is the relevant sufficient statistic for RPI feasibility.[6] This discounted sum needs to be large enough, in a manner we make precise. Interestingly, the discount factor is not the risk-free interest rate (which may be negative), but the marginal product of capital net of depreciation. The MPK is the rate at which government can trade "with mother nature" across time, while the rate at which it trades with households is the risk-free interest rate.

To provide additional insights behind the results, we specialize the analysis to a representative agent (RA) economy with separable utility that features a positive markup. We first ask whether the neoclassical efficient path constitutes an RPI. We show that for this, government debt is useful in "smoothing transfers." That is, debt reduces the need for the government to use lump sum taxes in the initial periods of the transition to a new steady state. Specifically, the government uses debt to finance an investment subsidy early on, and then services the debt by taxing the additional labor and profit income generated by the larger long-run capital stock. In this sense, government debt and capital investment are *complements* rather than substitutes. This contrasts the use of debt to "crowd in" capital via subsidies when capital is below the Golden Rule, versus the more familiar crowding out when the capital stock is too large, as in Diamond (1965).

We show that a feasible RPI exists if there is a non-zero markup and the inter-temporal elasticity of substitution is greater than the ratio of capital income to aggregate consumption in the initial economy. The two sides of the inequality show how a willingness to postpone consumption (the IES) and a relatively small aggregate income effect from higher interests (a small share of income due to interest payments) combine to satisfy the aggregate savings elasticity condition.

Note that the interest rate in the steady state of the RA economy will be positive (and equal to the inverse of the discount factor minus one). Hence, this also shows that a markup in and of itself may be sufficient for an RPI, even though the government must keep profits at least as large

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[5]The wedge between the MPK and the return on bonds can also be motivated by a liquidity premium, which we discuss in an appendix.

[6]This is potentially measurable in the data by integrating the discounted impulse response of aggregate household wealth to an exogenous change in in the risk-free interest rate or, inversely, the response of interest rates to an exogenous change in government debt held by the public. It is also easily calculated in a calibrated model using the techniques of Auclert, Bardóczy, Rognlie and Straub (2021).

as the initial level and cannot resort to lump-sum taxes to finance production subsidies.

After presenting the analytical results, we provide a simulation exercise to complement the analysis. Imposing Epstein and Zin (1989) preferences, using the income process of Krueger, Mitman and Perri (2016) and the historical data on $r - g$ in the U.S, we find scope for Robust Pareto Improving policies for a wide range of debt policies and for policies with and without capital crowding in. Our baseline experiment considers a Pareto-improving constant-K fiscal policy that starts at the laissez-faire equilibrium and slowly increases debt to 60% of output, the average observed in U.S. data over the last half-century. A second fiscal policy plan we consider consists of the same debt path as the baseline, but with capital increasing towards the Golden Rule. We find that this fiscal plan is also a feasible RPI and generates even larger welfare gains to all households, because in this case policy helps not only with risk sharing but also with efficient supply expansions. Debt is an essential part of this fiscal policy, as it provides the revenue that is required early on to finance the subsidies for the capital expansion. We do find, however, that seigniorage revenue from bonds has limits and features a Laffer curve: more debt increases interest rates and therefore the relative cost for servicing the debt. In our calibration, the upper bound on debt for Pareto improving fiscal policies is about twice the level of output.

1.1 Related Literature

This paper is part of a fast-growing recent literature exploring fiscal policy in environments with persistently low risk-free interest rates. Mehrotra and Sergeyev (2020) use a sample of advanced economies to document that $r - g$ is often negative, and, they develop a model to study the implications of this finding for debt sustainability.[7] Blanchard (2019)'s presidential address to the American Economics Association gave a major stimulus to the question of debt sustainability under low interest rates. Other recent papers are Bassetto and Cui (2018); Reis (2020); Brunnermeier, Merkel and Sannikov (2020); Ball and Mankiw (2021); and Barro (2020). Several of these papers focus on aggregate risk and build on Bohn (1995). Our paper incorporates features of this previous work such as borrowing constraints and the potential role of markups in opening a wedge between the interest rate and the marginal product of capital. However, our focus is on designing Pareto improving policies in the presence of individual heterogeneity and incomplete markets, as in the Bewely-Huggett-Aiyagari tradition, and on the role played by $r < g$.

Our work also contributes to the literature studying the effects of fiscal policies in models with heterogeneous agents. Heathcote (2005) shows the failure of Ricardian equivalence in this class of models: temporary tax cuts financed with public debt tend to increase consumption and output because they give households that are at the borrowing constraint extra resources, which

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[7]See also Mauro and Zhou (2021) and Jordà, Knoll, Kuvshinov, Schularick and Taylor (2019).

are spent. Heathcote, Storesletten and Violante (2017) study optimal labor tax progressivity in this environment and illustrate sharply the tradeoff between insurance and incentives motives in response to the tax system. Dyrda and Pedroni (2020) study the optimal tax system in a quantitative version of the idiosyncratic risk model. Krueger, Ludwig and Villalvazo (2021) consider an overlapping generations model in which agents face idiosyncratic risk in the final period of life. They evaluate the tradeoffs for general Pareto weights on different generations of a tax on capital that reduces income risk but potentially exacerbates inter-generational inequality. Also recently, Bhandari, Evans, Golosov and Sargent (2020) have explored optimal fiscal and monetary policy within the context of the heterogeneous agent model with nominal rigidities and aggregate shocks.[8] All of these papers focus on a utilitarian welfare criteria, and do not analyze the implications of $r < g$. In contemporaneous work, Kocherlakota (2021) studies the role of public debt bubbles for government deficits and expected consumption in models of heterogeneous agents that face tail risks, but abstracts from Pareto improvements and dynamic efficient environments.

As in any model in which seigniorage (or a liquidity premium) plays an important fiscal role, private households' willingness to hold additional government bonds without a large increase in the interest rate is key. The empirical literature on whether and to what extent government borrowing increases the interest rate is challenged by identification concerns and has produced results with no clear consensus. See, for example, the survey papers by Bernheim (1987) and Seater (1993), which examine the empirical evidence and seem to draw opposing conclusions on Ricardian equivalence.

Our RPI welfare metric is a potentially high hurdle and is not introduced as a replacement for the traditional Pareto or utilitarian criteria.[9] That said, the limited knowledge required on idiosyncratic preferences and risks make it a reassuringly robust threshold when feasible.[10] Given that income and wealth differences persist across generations (Chetty, Hendren, Kline and Saez, 2014), that some agents have limited access to financial markets (Braxton, Herkenhoff and Phillips, 2020), and that agents may value inter-temporal tradeoffs differently (Krusell and Smith, 1998), working through expanding all budget sets is an attractive approach when feasible.

The paper proceeds as follows: Section 2 describes the environment; Section 3 formally defines a Robust Pareto Improvement and provides conditions for when an RPI can be implemented in equilibrium; Section 4 provides the main analysis of how RPI fiscal policies work and derives

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[8]See also Le Grand and Ragot (2020). Other recent papers that have studied the implications of transfers and government debt in heterogeneous agent models with price rigidities are Oh and Reis (2012) and Hagedorn, Manovskii and Mitman (2019).

[9]Our focus on Pareto-improving policies rather than policies that maximize a utilitarian metric has an antecedent in Werning (2007), who explores Pareto-efficient tax policies in a Mirrelesian environment. See also Hosseini and Shourideh (2019).

[10]While not necessary for the RPI metric, for tractability we zero wealth effect on labor supply, as in the well known "GHH" preferences of Greenwood, Hercowitz and Huffman (1988).

a sufficient statistic for implamentability; Section 5 discusses a representative agent economy example in which we can derive the sufficient statistic from primitives of preferences and technology; Section 6 provides numerical examples; and Section 7 concludes.

2 Environment

The model hews closely to the canonical environment of Aiyagari (1994). We augment this framework with a government that issues debt, sets linear taxes on factor payments, and rebates back to households any fiscal surplus via lump-sum transfers. In many ways, however, our environment is more general. We allow for permanent differences in the income process or preferences across households. The framework also allows for product market markups, driving a wedge between the marginal product of capital and the return on risk-free bonds. We do impose one assumption on preferences—namely, that there is no wealth effect on labor supply, which greatly simplifies the impact of a change in interest rates on aggregate labor supply .[11]

We suppress exogenous growth in the text, but show in Appendix E how the model extends to growth in the usual straight forward way (given homothetic preferences). As a rule of thumb, the key condition $r < 0$ for an interest rate r in the baseline set-up is replaced with the corresponding $r < g$, where g denotes the constant exogenous growth rate of labor-augmenting productivity.

2.1 Households

Each household, from a measure-one continuum and indexed by $i \in [0, 1]$, draws an idiosyncratic labor productivity $z_t^i \geq 0$ at time t. We do not impose that households face the same stochastic process for idiosyncratic risk. That is, some households may face a permanently lower level of productivity or additional risk. Below we impose a cross-sectional independence restriction that rules out aggregate productivity risk.

If the household provides $n_t^i \geq 0$ units of labor, it receives $w_t z_t^i n_t^i$ in labor earnings; w_t is the equilibrium wage rate per efficiency unit of labor. Without loss of generality, we assume the firm pays labor taxes.

A household may also receive profit income. We model this as payment to entrepreneurial talent, which, like labor productivity, is an endowment that may follow a stochastic process. Let π_t^i denote household i's return to entrepreneurial talent. Define aggregate household profit income as $\Pi_t = \int \pi_t^i di$ and household i's share as $\theta_t^i \equiv \pi_t^i/\Pi_t$. Household i faces a potentially stochastic process for θ_t^i that determines its share of aggregate profits, with the restriction that

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[11]Our conceptual approach, however, can be extended to preferences with wealth effects easily. In this extension, the elasticity of the aggregate labor supply will matter as well.

$\theta_t^i \in [0, 1]$ and $\int \theta_t^i di = 1$ for all t.

At the start of period t, the household has a_t^i units of financial assets, which receive a risk-free return $(1 + r_t)$ in period t. Letting T_t denote lump-sum transfers from the government, which are uniform across i, the household's budget constraint is

$$c_t^i + a_{t+1}^i \leq w_t z_t^i n_t^i + \theta_t^i \Pi_t + (1 + r_t) a_t^i + T_t,$$

where c_t^i is consumption in period t.

Households are subject to a (potentially idiosyncratic) borrowing constraint $a_t^i \geq \underline{a}^i$ for all t. The fact that some households may have a tighter constraint than others captures the possibility that access to financial markets may be heterogeneous. Let $\underline{a} \equiv \inf_i \underline{a}^i$ denote the loosest borrowing constraint faced by households.[12]

The main restriction on preferences is the absence of a wealth effect on labor supply, as in the well known "GHH" preferences of Greenwood, Hercowitz and Huffman (1988). In particular, let $x^i(c, n) \equiv c^i - v^i(n)$ for some convex function v^i. We write preferences recursively as $V_t^i = \phi^i(x_t^i, h_t^i(V_{t+1}^i))$, where V_t^i is household i's value and h_t^i represents a certainty equivalent operator over idiosyncratic shocks $\{z_{t+1}, \theta_{t+1}\}$, conditional on z_t, θ_t and the household's stochastic process for its shocks. This notation nests both standard "CRRA" utility as well as the recursive utility of Kreps and Porteus (1978) and Epstein and Zin (1989). We incorporate the latter to explore the different roles of risk aversion and inter-temporal elasticity of substitution in the feasibility of a Pareto improvement.

The idiosyncratic state variables for an individual household are $s \equiv (a, z, \theta)$, and the aggregate states are the (perfect foresight) sequences for factor prices $\{w_t, r_t\}$, aggregate profit income $\{\Pi_t\}$, and transfers $\{T_t\}$. The household's problem can be written as follows:

$$V_t^i(a, z, \theta) = \max_{a' \geq \underline{a}^i, n \in [0, \bar{n}^i], c \geq 0} \phi^i(x^i(c, n), h_t^i(V_{t+1}^i(a', z', \theta'))) \tag{1}$$

$$\text{subject to: } c + a' \leq w_t z n + \theta \Pi_t + (1 + r_t) a + T_t.$$

We assume that the preference specification is such that all households value more consumption today and in the future (that is, ϕ^i is strictly increasing in x^i and in the continuation values, V_{t+1}^i).[13] Note that as preferences can vary across households, we can accommodate distinct la-

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[12]Below we assume that the borrowing constraint is always above the natural borrowing limit. See Bhandari, Evans, Golosov and Sargent (2017) and Heathcote (2005) for a discussion on the role of such ad-hoc limits in breaking Ricardian equivalence.

[13]We note that it is possible to generalize this and accommodate some hand-to-mouth households. In that case, we could consider the aggregator $\phi^i(x, h) = \phi^i(x)$ for some household i. This corresponds to a household that does not value future consumption (it has a discount factor equal to 0). As a result, this household does not save and consumes its entire disposable income every period.

bor supply elasticities. The framework also nests the classic Aiyagari (1994) model with inelastic labor supply.[14]

Assuming an interior labor supply decision, household i's first-order condition with respect to labor is

$$v_i'(n_t^i) = w_t z_t^i.$$

This implies a policy function $n_{i,t}^*(z)$, where the subscript t captures the equilibrium wage at period t.[15]

Similarly, we let $a_{i,t}^*(a, z, \theta)$ and $c_{i,t}^*(a, z, \theta)$ denote the optimal saving and consumption policy functions, respectively. The aggregate stock of savings chosen in period t and carried into period $t+1$ is

$$A_{t+1} \equiv \int a_{i,t}^*(a_t^i, z_t^i, \theta_t^i)di.$$

We now state our independence assumption. Let $z_t \equiv \{z_t^i\}_{i \in [0,1]}$ denote the assignment of productivity across households at time t. Let

$$N(w_t, z_t) \equiv \int z_t^i n_{i,t}^*(z_t^i)di = \int v_i'^{-1}(w_t z_t^i)di.$$

We make the assumption that N is independent of z_t. This is a generalization of the typical assumption that v is common across households and that z is i.i.d. across i and t. The current environment requires only that aggregate labor supply be independent of the distribution; this assumption is weaker than assuming that households are ex ante identical.[16]

2.2 Firms

The representative firm has a standard constant-returns technology given by $F(k, l)$, where k is capital and l effective units of labor. We impose that F is strictly increasing and concave on both arguments, twice differentiable in l, k, and satisfies Inada conditions.

Firms hire labor and rent capital in competitive markets at rates r_t^k and w_t, respectively. Let τ_t^n and τ_t^k denote linear taxes on factor payments for labor and capital, respectively.

Firms may have market power in the product market. We introduce the potential for mar-

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[14]This can be achieved by setting $v^i = 0$. In this case, the labor supply decision is not interior and the corresponding first-order condition below does not hold.

[15]As we will see below, the Frisch elasticities of labor supply, encoded in the function v_i are not important for the analysis (beyond determining the initial equilibrium allocation), as the policies that we explore maintain a constant after-tax wage.

[16]For example, households could belong to one of J types, each with non-trivial measure. Then, within a type, we can assume that the law of large numbers holds, and the aggregate is simply a weighted average across types.

ket power for two primary reasons. One is to ensure our analysis is robust to the presence of mark-ups, which appear to be a feature of the data. Second, it introduces a wedge between the marginal product of capital and the return on household savings. There are several alternative interpretations of why there may be a difference in the marginal product of capital and the return to bonds, even in the absence of risk premia, that we discuss in the next subsection.

For simplicity, we assume that firms charge a price that is a constant markup over marginal cost. Let $\mu \geq 1$ be the ratio of price to marginal cost. The representative firm's first-order conditions imply that

$$F_k(K_t, L_t) = \mu(1 + \tau_t^k)r_t^k$$
$$F_l(K_t, L_t) = \mu(1 + \tau_t^n)w_t.$$

where K_t and L_t represent the aggregate capital and labor demands.

Firm (pre-tax) profits are given by

$$\widetilde{\Pi}_t = F(K_t, L_t) - (1 + \tau_t^k)r_t^k K_t - (1 + \tau_t^n)w_t L_t = \left(\frac{\mu - 1}{\mu}\right)F(K_t, L_t).$$

where the last equality follows from constant returns.

Profits are taxed by the government at rate τ_t^π, so after-tax profits are $\Pi_t = (1 - \tau_t^\pi)\widetilde{\Pi}_t$. We can think of the representative firm hiring a bundle of entrepreneurial talent that is in constant aggregate supply at after-tax price Π_t.

For some of the analysis that follows, it will be useful to distinguish cases when capital is above or below the "Golden Rule" rule. For a given N, we define the Golden Rule capital level $K^\star$, by $F_K(K^\star, N) = \delta$.

2.3 Financial Intermediaries

We assume that the capital is owned by financial intermediaries.[17] Such intermediaries are competitive. They borrow from the households at rate r_t and, in turn, rent capital to firms at r_t^k and invest in government bonds at rate r_t^b. Capital depreciates at rate δ. Competition in the intermediary market ensures the following equilibrium condition at all t:

$$r_t = r_t^b = r_t^k - \delta.$$

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[17] As is usually the case, making this assumption is not crucial. We could have equivalently assumed that the capital is owned directly by firms, which finance capital purchases with risk-free bonds issued to households.

Given the first equality, in what follows, we drop the distinction between r_t and r_t^b. Note that there is also no maturity mismatch on the intermediaries' balance sheet.

As noted above, the potential presence of a mark-up implies that the return on financial assets may differ from the marginal product of capital. There are several alternative (or additional) reasons for such a wedge in practice. Uncertainty regarding the return to physical investment would potentially impose a risk premium on the required rate of return to capital. As noted above, we are abstracting from such risk in order to transparently highlight the novel aspects of our analysis.

Even under perfect foresight, there may be additional reasons for a wedge between the marginal product of capital and the risk-free interest rate. One alternative pursued by, for example, Ventura (2012), is to introduce firm-level borrowing constraints. A second alternative is that government bonds provide "liquidity services" relative to the return on physical capital. This latter possibility can be readily introduced by modifying the intermediaries problem.

Suppose that a competitive intermediary receives flow return $r^k - \delta$ from holding physical capital and $r^b + \rho$ from holding government bonds, where ρ is the additional (pecuniary) return provided by government bond's "liquidity." The value of ρ may depend on the aggregate stock of government bonds (as suggested by Krishnamurthy and Vissing-Jorgensen, 2012), but is taken as given by an individual intermediary. Equilibrium requires $r^k - \delta = r^b + \rho$.

2.4 Government

The government's policy consists of a sequence of taxes $\{\tau_t^n, \tau_t^k, \tau_t^\pi\}$, as well as a sequence of one-period debt issuances, $\{B_t\}$. The lump-sum transfers T_t are such that the sequential budget constraint holds at all periods:

$$T_t \leq \tau_t^n w_t L_t + \tau_t^k r_t^k K_t + \tau_t^\pi \widetilde{\Pi}_t + B_{t+1} - (1 + r_t)B_t. \tag{2}$$

Note that we allow for the government to potential dispose of resources freely by writing the constraint as an inequality.

2.5 Resource Constraint and Market Clearing

Market clearing in the asset market requires $A_t = K_t + B_t$. Market clearing in the labor market requires $L_t = N_t$; recall that N_t is aggregate efficiency units of labor supplied by households. Using these, the aggregate resource constraint is

$$C_t \equiv \int c_{i,t}^* di \leq F(K_t, N_t) - K_{t+1} + (1 - \delta)K_t.$$

Definition 1 (Equilibrium Definition). *Given an initial distribution of household assets and idiosyncratic shocks $\{a_0^i, z_0^i, \theta_0^i\}_{i \in [0,1]}$ and a fiscal policy $\{B_t, \tau_t^n, \tau_t^k, \tau_t^\pi, T_t\}_{t \geq 0}$, an equilibrium is a sequence of quantities $\{A_t, K_t, N_t, \Pi_t\}_{t \geq 0}$, and prices $\{r_t, r_t^k, w_t\}_{t \geq 0}$ such that: A_t and N_t are the aggregate stock of savings and the aggregate labor supply consistent with household optimization given prices and transfers, Π_t is the aggregate after-tax profits, K_t and N_t are the aggregate capital and labor demands consistent with firm optimization given prices and taxes, the sequential government budget constraint is satisfied, the aggregate resource constraint holds, $r_t^k = r_t + \delta$, and the asset market clears.*

We define a stationary equilibrium to be an equilibrium in which all sequences are constant over time.[18]

3 Robust Pareto Improvements

In this section we introduce and discuss our welfare metric, "Robust Pareto Improvements" (RPI). We then provide necessary and sufficient conditions for a class of RPI to be implementable as an equilibrium.

3.1 A Robust Welfare Metric

Given idiosyncratic states, a household's welfare is determined by sequences of (after tax) factor prices, $\{w_t\}$ and $\{r_t\}$, aggregate profits, $\{\Pi_t\}$, and transfers $\{T_t\}$. These are the equilibrium objects that appear in the budget set of the household problem (1). With this in mind, we define what we mean by a "robust" Pareto improvement:

Definition 2. *Consider two sequences of factor payments $\{w_t^i, r_t^i, \Pi_t^i\}_{t \geq 0}$ and transfers $\{T_t^i\}_{t \geq 0}$, with $i = A, B$. We say sequence A generates a* Robust Pareto Improvement *(RPI) over sequence B if it expands budget sets for every agent at every time and every state:*

$$w_t^A \geq w_t^B, \ \Pi_t^A \geq \Pi_t^B, \ r_t^A \geq r_t^B, \ T_t^A \geq T_t^B - (r_t^A - r_t^B)\underline{a} \ \text{for all } t \geq 0,$$

with at least one strict inequality.

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[18]In the analysis that follows, we will assume that such an stationary equilibrium exists. Note that this may require additional assumptions on the stochastic processes for labor productivity and the profit share as well as on their initial cross-sectional distribution. See Açıkgöz (2018), Light (2018), and Achdou, Han, Lasry, Lions and Moll (2021) for results on the existence and uniqueness of stationary equilibria in Bewley-Huggett-Aiyagari models.

From the sequential budget set governing the household's problem (1), we see that the consumption possibility set is weakly increasing in w and Π. If $a \geq 0$, it is also weakly increasing in r. However, households with negative positions (debt) are worse off if r increases. The fact that $T_t^A - T_t^B \geq -(r_t^A - r_t^B)\underline{a}$ ensures that additional lump-sum transfers are large enough to make debtors weakly better off and strictly better off if $a_t^i > \underline{a}$. From every household's perspective, resources are weakly greater at every t and at every idiosyncratic state, and they are strictly greater for at least a positive measure of households at some t.

The term "robust" is meant to highlight that limited knowledge is required about idiosyncratic preferences or sources of income. All that is needed to ensure an individual prefers a fiscal policy is that a larger budget set is a good thing for the consumer. In particular, how an individual values inter-temporal or inter-state trades plays no role.

It is instructive to clarify how this metric is distinct from some well known alternatives and how it rules out prominent policies studied in the literature.

For example, in the classic analysis of government debt in an incomplete markets setting, Aiyagari and McGrattan (1998), a government issues bonds, transfers the proceeds to households, and then levies lump-sum taxes to pay interest on the debt. Unconstrained households can save the transfers in anticipation of the taxes, while those constrained can effectively bring future income forward. This policy effectively relaxes the borrowing constraint (as in Woodford (1990)). From a "period-0" perspective, this may represent a welfare gain to households. However, the fact that future taxes increase violates the conditions for an RPI. Moreover, the introduction of government debt may crowd out capital and reduce the equilibrium wage, representing another violation of RPI. Consider a household that earns only labor income and lacks access to financial markets, which is not an unrealistic description of some households in the data. They may be strictly worse off in the Aiyagari-McGrattan experiment, but not under an RPI.

Another well-known paper is Dávila et al. (2012). That paper characterizes constrained efficient equilibria under a utilitarian metric. The focus of the analysis is whether alternative consumption or labor supply decisions by households could alter equilibrium factor prices in such a way as to raise the utilitarian objective function. However, it may be the case, for example, that the efficient equilibrium features an decrease in the interest rate and an increase in the wage, which involves a trade off that violates the definition of an RPI.

Perhaps the most common metric for evaluating policy is the traditional Pareto criteria, in which every household's expected discounted utility at time zero weakly increases, with a strict increase for at least one. For example, the welfare consequences of government debt is evaluated under the Pareto criteria in Diamond (1965), who highlights both the impact on welfare of both taxes as well as the associated change in factor prices due to the crowding out of capital. Similarly, Samuelson (1975) uses a Pareto criteria to evaluate social security policies that reduce resources

while young in exchange for transfers while old. Similarly, several papers explore Pareto improvements in an incomplete markets setting (see, for example, Krueger et al. (2016), Hosseini and Shourideh (2019), and Boerma and McGrattan (2020)). Pareto-improving policies in this setting may involve, for example, better insurance, so that income increases in some states at the expense of others. Again, these tradeoffs may be desirable given a particular set of preferences and beliefs, but do not represent RPIs.

The role of introducing the concept of an RPI is not to displace either the utilitarian metric or the standard Pareto measure. These metrics have many compelling features. The advantage of the RPI metric is we do not need to take a strong stand on preferences, the nature of idiosyncratic risk, or heterogeneity in either of these across households when evaluating policies. Of course, expanding all budget sets at all dates and times is potentially a high hurdle for policy analysis. This begs the question as to whether and when an RPI is attainable in equilibrium given the limited fiscal tools available to the government.

3.2   Feasibility of an RPI

In this subsection, we derive necessary and sufficient conditions for a fiscal policy to be consistent with the restrictions imposed by equilibrium. That is, given a limited set of taxes, we describe the allocations that the government can implement as equilibria.

We assume that the economy starts at a stationary equilibrium, which potentially may have an amount of government bonds outstanding as well as distortionary taxes. As is clear from the definition of an RPI, we could also start from a non-stationary equilibrium, but in that case all comparisons would be relative to the initial "reference" sequence of factor prices.

Let (w^o, r^o, Π^o) denote the wage, interest rate, and aggregate profits in the initial stationary equilibrium, and let (N^o, K^o) denote the associated aggregate labor supply and capital stock. Let B^o denote government debt in the initial equilibrium, financed by $\{\tau^{no}, \tau^{ko}, \tau^{\pi o}\}$. For simplicity, we assume that there are zero lump-sum transfers in the initial equilibrium, $T^o = 0$, and tax revenue equals $r^o B^o$ (that is, the government budget constraint holds with equality).

Starting from this equilibrium, consider that the government unexpectedly announces a new fiscal policy. That is, in period $t = 0$, the government announces a sequence of debt issuances, taxes, and transfers $\{B_{t+1}, \tau_t^n, \tau_t^k, \tau_t^\pi, T_t\}_{t \geq 0}$. After the announcement, there is perfect foresight. Given the new policy, households and firms re-optimize. Consider a new equilibrium that arises, with aggregate quantities $\{A_t, K_t, N_t, \Pi_t\}$, and prices $\{r_t, r_t^k, w_t\}$. The level of period-0 capital and debt, as well as r_0, are inherited from the initial equilibrium, so $K_0 = K^o$, $B_0 = B^o$, and $r_0 = r^o$.

We restrict attention to policies that keep after-tax wages and profits unchanged from the initial equilibirium:

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Definition 3. *A "constant wage and profit policy" ensures $w_t = w^o$ and $\Pi_t = \Pi^o$ for all $t \geq 0$.*

Under a constant wage a profit policy, no agent experiences a change in labor or profit income at each t and idiosyncratic state (z_t^i, θ_t^i). This restriction is useful for two reasons. One is that the constant wage ensures the labor market clears at the original employment N^o, regardless of the elasticity of labor supply.[19] The second is that it allows us to keep the interplay of government debt issuance and changes in the interest rate in the foreground.

In period zero, each household re-optimizes its consumption-saving policy to incorporate a new sequence of interest rates and transfers, $(\boldsymbol{r}, \boldsymbol{T}) \equiv \{r_t, T_t\}_{t \geq 0}$ while maintaining the remaining factor incomes constant at (w^o, Π^o). Starting from the initial stationary equilibrium in period 0, we define the following functions:

Definition 4. *Let $\mathcal{A}_{t+1}(\boldsymbol{r}, \boldsymbol{T})$ denote the aggregate household assets at the end of period t generated by the households' optimization given $w_t = w^o$ and $\Pi_t = \Pi^o$ for all $t \geq 0$. The associated aggregate consumption function is*

$$C_t(\boldsymbol{r}, \boldsymbol{T}) \equiv w^o N^o + \Pi^o + (1 + r_t)\mathcal{A}_t(\boldsymbol{r}, \boldsymbol{T}) - \mathcal{A}_{t+1}(\boldsymbol{r}, \boldsymbol{T}) + T_t.$$

That is, if $a_{i,t}^*(a, z, \theta)$ and $c_{i,t}^*(a, z, \theta)$ denote household i's policy functions in the new equilibrium, then $\mathcal{A}_{t+1} = \int a_{i,t}^*(a_t^i, z_t^i, \theta_t^i)di$ and $C_t = \int c_{i,t}^*(a_t^i, z_t^i, \theta_t^i)di$. These mappings of the sequence of interest rates and transfers to the sequence of aggregate household assets and consumption summarizes how a fiscal policy affects aggregate saving behavior in equilibrium.

The standard "primal" approach in the Ramsey taxation literature is to restrict attention to the set of allocations that can be achieved in a competitive equilibrium by feasible fiscal policies, replacing taxes and prices using equilibrium conditions. We follow a similar approach, with the one caveat that we cannot rely on a representative consumer's Euler equation to solve out the interest rate. In it's place, we include the restriction imposed by household optimality implied by the mapping $\mathcal{A}_t$.

We say that the sequence $\{r_t, T_t\}_{t \geq 0}$ is *feasible* if there is a fiscal policy $\{B_t, \tau_t^n, \tau_t^k, \tau_t^\pi, T_t\}_{t \geq 0}$ with $B_o = B^o$ such that a competitive equilibrium with quantities $\{A_t, K_t, N^o, \Pi^o\}_{t \geq 0}$ and prices $\{r_t, r_t^k = r_t + \delta, w^o\}_{t \geq 0}$ exists, where $K_0 = K^o$ and $A_0 = A^o$. We have the following result:

Lemma 1. *A sequence of interest rates and transfers, $(\boldsymbol{r}, \boldsymbol{T})$, is feasible if and only if there exists a non-negative sequence $\{K_t\}_{t \geq 0}$ and a sequence $\{B_t\}_{t \geq 0}$, with $K_0 = K^o$ and $B_0 = B^o$, such that for all $t \geq 0$:*

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[19]For this, we are using the assumption of zero wealth effect on labor supply.

(i) $\mathcal{A}_{t+1}(\boldsymbol{r}, \boldsymbol{T}) = B_{t+1} + K_{t+1},$

(ii) *and*

$$B_{t+1} - (1 + r_t)B_t - T_t \geq F(K^o, N^o) - F(K_t, N^o) - (r^o + \delta)K^o + (r_t + \delta)K_t - r^o B^o. \qquad (3)$$

Proof. All proofs are in Appendix C. □

This lemma provides necessary and sufficient conditions for the sequences $\{r_t, T_t, B_t, K_t\}$ to be consistent with equilibrium. Household optimality is ensured by the definition of $\mathcal{A}$. Asset market clearing is condition (i) of the lemma. Condition (ii) combines firm optimality and government budget balance, and is discussed next. The aggregate resource constraint (goods market clearing) holds by Walras law. Note that these conditions must be met by *any* constant wage and profit fiscal policy, not just those that result in an RPI.

The left hand side of condition (3) is the revenue raised from net debt issuance minus any lump sum transfers. The right hand side is the fiscal cost of the subsidies necessary to keep wages and profits constant. In particular, simple accounting implies that government revenue raised by taxing firms is total output minus the firm's after-tax payments to households:

$$\text{Taxes paid by firms } = F(K_t, N^o) - \Pi^o - w^0 N^o - (r_t + \delta)K_t, \qquad (4)$$

where we use the fact that after-tax wages and profits are unchanged. In the initial equilibrium, we have a similar expression, where tax revenue is used to pay interest on the initial debt, B^o. Hence,

$$r^o B^o = F(K^o, N^o) - \Pi^o - w^0 N^o - (r^o + \delta)K^o. \qquad (5)$$

Using this expression to substitute out $\Pi^o + w^0 N^o$ in (4) and converting from revenues to subsidies by changing sign, we obtain the right-hand side of (3) as tax subsidies to firms. The inequality follows from the fact that the government is free to dispose of any fiscal surplus it does not choose to lump-sum rebate.

The implications of condition (3) on whether an RPI is feasible will be the focus of the next sections. At this stage, we flag three immediate consequences that will play prominent roles in what follows. First, an increase in r_t (for a given K_t) increases the right-hand side of (3), tightening the constraint. Higher interest rates are costly, as the government needs to subsidize capital to avoid a reduction in firms' demand for the factor. Second, an increase in K_t (for a given r_t) reduces the right hand side of the (3) in the presence of a positive markup as $F_K > r + \delta$, relaxing the constraint. However, increasing K_t may require an increase in interest rates, to encourage the

household sector to save. Finally, $r_t < 0$ implies $r_t B_t < 0$ for $B_t > 0$, which is the left-hand side of (3) . This captures the fact that negative interest rates are a potentially important source of revenue for a government that borrows.

A convenient feature of Lemma 1 is that the feasibility of sequence of interest rates and transfers is solely determined by aggregates. No additional information is needed, despite the potentially complicated nature of the policies necessary to keep the wage and profits constant and the potentially rich sources of heterogeneity underlying the aggregate saving and consumption functions.

Walras law allows an alternative to Lemma 1 that involves the aggregate resource constraint. Aggregating the households' budget constraints, we have:

$$
\begin{aligned}
C_t &= w^o N^o + \Pi^o + (1 + r_t)\mathcal{A}_t - \mathcal{A}_{t+1} \\
&= F(K^o, N^o) - r^o(B^o + K^o) - \delta K^o + (1 + r_t)(K_t + B_t) - (K_{t+1} + B_{t+1}),
\end{aligned}
$$

where the second equality follows from asset market clearing and (5). Substituting into (3), we obtain:

Corollary 1. *A sequence of interest rates and transfers, (r, T) is feasible if and only if there exists a non-negative sequence $\{K_t\}_{t \geq 0}$ with $K_0 = K^o$ such that for all $t \geq 0$:*

$$
C_t(r, T) \leq F(K_t, N_0) + (1 - \delta)K_t - K_{t+1}. \tag{6}
$$

Corollary 1 reduces the question of feasibility to the existence of an investment sequence that "finances" the aggregate consumption generated by the policy.[20] The next section leverages Lemma 1 and Corollary 1 to explore the economics of engineering an RPI.

4   Robust Pareto Improving Policies

In this section we delve into the details of when and how fiscal policy can engineer a RPI. We begin with a simple policy in which the government issues bonds but keeps capital at a constant level. This exercise will allow us to focus in on how an increase in interest rates generates a welfare gain without changing aggregate resources. We then move to general policies in which the capital stock may evolve over time.

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[20] A reader may wonder why the initial level of debt, B^o, does not explicitly appears here while it did in Lemma 1. We note that B^o implicitly appears in the initial asset position of households A^o and thus affects the aggregate consumption function $C_t(r, T)$.

4.1 The Constant-K Policy

Let us first consider whether an RPI can be implemented with a constant level of capital. There are many nice features of such a policy. As we will see, one advantage of such a policy is that the feasibility of a RPI applies whether the economy is fully competitive, $\mu = 1$, or has markups, $\mu > 1$, and whether the capital is stock is above or below the Golden Rule level. A second feature is that it highlights the fact that an RPI is possible without changing total resources – all gains are derived from a better allocation of the same amount of output, while keeping in mind that every household has more resources available for consumption at every state and time.

For simplicity, in this section we assume that the economy is originally at the laissez-faire stationary equilibrium with zero taxes and transfers and $B^o = 0$. Now consider a policy in which the government permanently raises the after-tax return to savings to $r_t = r' > r^o$, for all $t \geq 1$. At the same time, the government sets T_t to its respective lower bound consistent with an RPI, $T_t = T' = -(r' - r^o)\underline{a}$.

To gain some intuition, let us suppose that the economy converges to a new stationary equilibrium with $\lim_{t\to\infty} \mathcal{A}_t(\{r', T'\}) = A_{ss} < \infty$.[21] Assuming that the stationary aggregate savings schedule is upward sloping with respect to the interest rate (as is the case in most of the applications of the Aiyagari model), we have that $A_{ss} > A^o = K^o$. Letting $B_{ss} = A_{ss} - K^o$ denote the long run supply of government bonds, we then have that $B_{ss} > 0$: the permanent increase in interest rates is associated with a permanent increase in government debt. Condition (3) requires that in the limit:

$$-r'B_{ss} \geq (r' - r^o)(K^o - \underline{a}). \tag{7}$$

Given that the right hand side is strictly positive, it is necessary for this RPI to be feasible that $r' < 0$: there must be "seigniorage" from bonds, and this seigniorage revenue must be greater than the right-hand side of (7).

It is helpful to explain the right-hand side of condition (7). The increase in the interest rate in the new equilibrium would raise the rental rate of capital and reduce the firm demand for the factor, all else equal. With a constant-K policy, the government must subsidize the return from renting capital to avoid this reduction in factor demand. Recall that r^{ko} is the rental rate in the initial equilibrium, with $r^o = r^{ko} - \delta$. In the new equilibrium, the government must set a capital subsidy, $\tau_t^k < 0$, such that firms pay the same rental rate as in the original equilibrium: $r^{ko} = (1 + \tau_t^k)(r' + \delta)$. As firms are paying the same after-tax rental rate, then $K_t = K^o$, and hence profits, wages, and total output remain unchanged. From the government budget constraint,

---

equation (2), the cost of this subsidy is $(r' - r^o)K_0$. The condition then tells us that the stationary revenue from bond issuances must be enough to cover the cost of the capital subsidy plus the cost of the transfer necessary to compensate borrowers for the increase in the rate.

Figure 1 depicts the steady-state tradeoff in the canonical capital market equilibrium diagram from Aiyagari (1994). The underlying calibration is provided in Section 6, but the qualitative features are fairly general. At each interest rate on the vertical axis r, the associated rental rate of capital is $r^k = r + \delta$. Holding labor supply constant, $N = N^o$, the downward-sloping red line traces out a capital demand equation from the firm's first-order condition $F_K(K, N^o) = \mu(r + \delta)$, where recall we assume the initial equilibrium has zero taxes.

Similarly, at each candidate r, A denotes the aggregate steady-state saving of households when the wage is fixed at w^o. These two curves intersect at the laissez-faire equilibrium interest rate r^o, which is the initial equilibrium. Note that in this parameterization, $r^o < 0$, which is the case of interest. The quantities reflected on the horizontal axis are normalized by $Y^o = F(K^o, N^o)$.

The fiscal policy subsidizes the rental of capital, so that firms are willing to rent K^o at any r. The width of the gray rectangle is $\Delta B/Y^o = \frac{A_{ss} - K^o}{Y^o}$, and its height is the interest rate at the new equilibrium; hence, its area is $-r'\Delta B/Y^o$. Starting from zero debt, this area is the left-hand side of (7).

The red rectangle has height $r' - r^o$, where r^o is the interest rate in the laissez-faire equilibrium. Its width is K^o/Y^o, where K^o is the capital stock in the laissez-faire equilibrium. The area of this rectangle is $(r' - r^o)K^o/Y^o$, which equals the subsidies necessary to keep capital at K^o. In this example, $\underline{a} = 0$, and hence this is the right-hand side of (7). Condition (7) tells that a necessary condition for *the RPI to be feasible* is that the area of the gray rectangle exceeds that of the red.

Note that this implies that feasibility is a tighter condition than being on the "upward sloping" portion of the debt Laffer curve. The steady state debt Laffer curve peaks when $-r * \Delta B$ is maximized; that is, when the gray rectangle achieves its maximum.[22] After that point, the increase in the interest rate dominates the additional debt issuance and seigniorage revenue declines. However, the fiscal cost, $\Delta r * K^o$, is strictly increasing in r, and hence the net revenue (seigniorage minus capital subsidy) for the constant-K policy peaks at a level of debt strictly below the peak of the debt Laffer curve.

The diagram restricts attention to the steady state, but contains important insights into the requirements for an RPI to be feasible. The first thing to note is that the level of the initial interest rate matters. That is, households must be willing to hold the economy's wealth at a low interest rate, reflecting a significant demand for precautionary savings.[23] Intuitively, and as we shall

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[22]Recent papers that focus on the debt Laffer curve include Bassetto and Sargent (2020) and Mian, Straub and Sufi (2022).

[23]For this policy, as we already mentioned, it is necessary that $r' < 0$, or else (7) cannot hold.

Figure 1: Net Resource Cost with Constant K
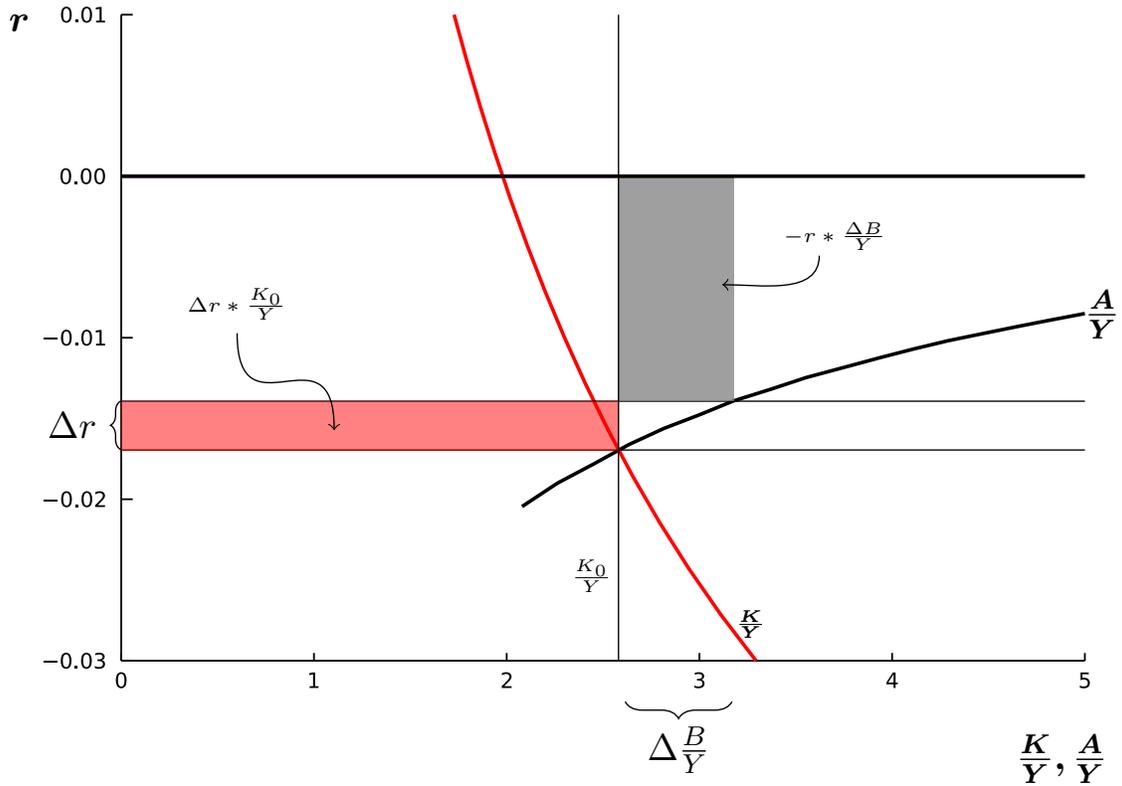
Note: This figure is a graphical depiction of the fiscal tradeoff from condition (7). All elements are normalized by the laissez-faire stationary equilibrium output $Y = Y^o$. The downward-sloping line K/Y represents the firm's demand for capital ($r = F_K/\mu - \delta$), and the upward-sloping line A/Y depicts aggregate household saving associated with the interest rate r and the initial wage w^o as well as the transfers generated by any fiscal surplus. The intersection is the initial laissez-faire stationary equilibrium. Fiscal costs are represented by $\Delta r * K^o/Y$, the area shaded in red, and seigniorage revenue by $-r * \Delta B/Y$, the area shaded in gray. In this example, policy holds capital at the initial laissez-faire capital stock.

see in detail in the calibration of Section 6, this will be the case if households face significant idiosyncratic risk and are patient and risk averse. The large demand for a safe store of value provides a source of seigniorage for the government.

Second, consumers must be willing to hold new debt without a sharp increase in the interest rate. That is, the elasticity of aggregate savings to r must be sufficiently large. The intuition is that the return to saving (Δr) cannot increase significantly in response to the issuance of ΔB, as the increase in the return to capital is the amount of subsidy necessary to keep capital constant. The elasticity of the interest rate to government debt is a primary concern when discussing the crowding out of capital. Here, it determines the amount of fiscal resources that must be dedicated to capital subsidies.

Note that the key elasticity is that of *aggregate* household savings. This echoes the point made in the previous section that household heterogeneity matters only as it determines the slope of the aggregate savings function. This elasticity can potentially be estimated using aggregate time series, and we survey some of the estimates from the literature in Section 4.2.3.

Third, conditional on the initial equilibrium, the feasibility condition is independent of the shape of the aggregate production function or the presence or size of a mark-up. This is because capital and labor do not change under the constant-K policy. The focus is purely on the shape of the households' aggregate saving function.

This raises an intriguing feature of the Pareto improvement. Aggregate output, consumption, and investment are all held fixed at the initial level, as $K_t = K^o$ and $N_t = N^o$. Yet every household faces a weakly bigger budget set and a strictly bigger one if $a_t^i > \underline{a}$. However, it cannot be the case that aggregate consumption increases. The key is that the higher interest rates induces enough households to reduce their consumption to offset the households that do increase consumption. This is why the elasticity of aggregate saving to the interest rate plays such a crucial role. Heuristically, those with high labor endowment states must be willing to postpone consumption because of the high return on saving. Those with low endowment states on average carry in higher precautionary savings, allowing them to consume more. On net, aggregate consumption remains constant, but it is distributed in a more beneficial way across idiosyncratic states.

The source of the welfare gains in this example has a clear antecedent in Samuelson (1958). In Samuelson, the young delay consuming their endowment in exchange for paper (money), and then trade the paper to the next generation when old. Here, the higher interest rate ensures that private households are willing to hold more government bonds, which they then trade to smooth consumption across states and time. While the sources of heterogeneity and risk are more general in the current set up, the main difference is that we are studying a production economy in which factor prices also affect welfare. In our case, it is not always the case that a robust Pareto improvement is feasible, despite $r^o < 0$, because the government needs enough

revenues for the capital subsidy, to guarantee that wages and profits do not fall.

Importantly, the source of welfare gains in our environment are distinct from other fiscal schemes in which an agent "pays in" or is taxed. That is, the requirements of an RPI rule out better insurance or reallocation via progressive taxation, tax-and-transfer insurance schemes, or a pay-as-you-go social security system. In the words of Samuelson (1958), the willingness to hold government bonds at low interest rates is a substitute for the "social coercion" of tax and transfer schemes.

Figure 1 depicts the steady state tradeoff faced by a government implementing the constant-K policy. Along the transition, Lemma 1 condition (3) requires (for a constant-K) that

$$\mathcal{A}_{t+1} - (1 + r_t)\mathcal{A}_t \geq -r^o K^o, \tag{8}$$

where we have imposed for additional simplicity that $\underline{a} = 0$ (and thus $T_t = -(r_t - r^o)\underline{a} = 0$) and used that $K_t + B_t = \mathcal{A}_t$. This inequality highlights that it is the response of aggregate savings at all periods to changes in interest rates that determines the feasibility of an RPI with a constant-K policy. Smaller short-run elasticities of aggregate savings make the condition (8) harder to satisfy even if it were to hold in the long-run (where the elasticity is potentially higher). The condition also highlights that a gradual approach may have a better chance of working with a constant-K policy: if the aggregate savings are very inelastic in the short-run, a permanent increase in r from the beginning may be infeasible as an RPI, while a gradual increase may work. In the simulation of Section 6 we provide an example of this, and of how debt issuance along the transition ensures (8) holds at all t, despite the relatively small short-run elasticity.

The constant-K policy is a useful benchmark to study RPIs because it is robust not only to rich household heterogeneity, but also to production elasticities and markups: the feasibility conditions for RPIs in (8) do not depend on these production-side details. However, if a constant-K policy cannot generate a feasible RPI, it may still be feasible for the government to adjust both K_t as well as B_t when implementing a policy. We turn to this more general policy problem next.

4.2   General Policies

In this subsection, we study more general policies, which may involve changes to the capital stock, that can achieve an RPI. We derive sharper conditions for feasibility and consider in more detail the transition path. As in the constant-K policy, we find that the elasticities of aggregate savings continue to be a main determinant for feasibility.

Our guide will be Lemma 1 and Corollary 1. As in the case discussed above, a common theme will be ensure that aggregate consumption chosen by households does not increase "too much," despite the fact that budget sets expand. Again, the countervailing force will be an aggregate

willingness to save induced by an increase in interest rates. We consider two cases in turn, one in which the initial equilibrium has over accumulated capital (that is, capital is beyond the Golden Rule level) and then the converse case. Recall that the Golden Rule capital level, $K^\star$, is defined by $F_K(K^\star, N^o) = \delta$.

4.2.1 Capital above the Golden Rule

If capital is above the Golden Rule, there is a relatively straightforward path to an RPI. The approach builds on Diamond (1965), substituting government bonds as a replacement for the over-accumulated capital. We are in an environment with potentially richer idiosyncratic heterogeneity and have a stricter welfare metric, and hence need to worry about changes in factor prices as capital is reduced. In particular, we cannot trade off lower wages against higher interest rates, or higher consumption when old with lower consumption when young. However, Corollary 1 has already done most of the work regarding the feasibilty of an RPI in this environment.

To start, first note that the capital sequence $\{K_t\}_{t=0}^\infty$ with $K_0 = K^o$ and $K_t = K^\star$ for $t \geq 1$ satisfies the resource constraint (6), with a strict inequality *at the original interest rates and transfers*, $r = r^o$ and $T = T^o = \{0, 0, \dots\}$. This is because, at the same interest rates and transfers, the households' problems have not changed, aggregate consumption remains as in the original, $C^o = F(K^o, N^o) - \delta K^o$ for all $t \geq 0$, but initial capital above the golden rule implies that a lower investment increases net resources at all dates.[24] From Corollary 1, this strict inequality means there are surplus resources at every date with this new capital sequence.

We can use the language of Lemma 1 to reinterpret this result using the government budget constraint. In particular, for this case, the government issues an amount of bonds $B_1 = K^o + B^o - K^\star > B^o$ in period 0, and then sets $B_t = B_1$ thereafter. The government policy must guarantee that r_t and T_t do not change, which requires an increase in the tax on capital (to reduce the firms' demand for capital), and an increase in the subsidy to labor and profits (to compensate labor and profits for the fall in capital). The strict inequality in (6) translates into a strict inequality in (3): the increase in revenue from the capital tax more than compensates for the cost of the subsidies, and the government runs a strictly positive budget surplus at all times, which it discards.

Although we have uncovered a policy where the government runs a surplus (that it discards), this policy does not yet constitute an RPI as defined (as interest rates and transfers have not changed). A natural enhancement is to lump-sum rebate the surplus, which would then constitute

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[24]For $t = 0$, the resource condition (6) is $C^o + K^\star < C^o + K^o = F(K^o, N^o) + (1 - \delta)K^o$, as $K^\star < K^o$. For $t \geq 1$,

$$C_t(r^o, T^o) + K_{t+1} = C^o + K^\star = F(K^o, N^o) - \delta K^o + K^\star < F(K^\star, N^o) - \delta K^\star + K^\star = F(K_t, N^o) + (1 - \delta)K_t,$$

where the first equality uses the fact that household's problem has not changed; the second equality uses goods market clearing in the original equilibrium; the strict inequality uses the fact that $F_K(K^o, N^o) < \delta$ for $K \in (K^\star, K^o]$; and the final equality uses $K_t = K^\star$ for all $t \geq 1$.

23

an RPI. The question is whether the asset market can still clear at the original interest rate. If a continuity assumption on how aggregate consumption (or, equivalently, aggregate savings) responds to transfers is satisfied, the answer is yes.

Specifically, consider a sequence of transfers $\widehat{T} = (T_0, T_1, \dots)$ with $T_t \geq 0 = T^o$ for all $t \geq 0$. Let v be a positive scalar that governs the magnitude of the change in transfers in the direction $\widehat{T}$. The following lemma states that there exists a sequence $\widehat{T}$ and magnitude v that can be the basis for an RPI:

Lemma 2. *Suppose $K^o > K^\star$ and consider a sequence $\hat{T} = (T_0, T_1, \dots)$ with $T_t \geq 0 = T^o$ for all $t \geq 0$ with at least one inequality strict. Suppose there exists an interval $(0, \varepsilon)$ and a $M > 0$ for which the following regularity condition holds:*

$$|C_t(r^o, T^o + \widehat{T}v) - C_t(r^o, T^o)| \leq Mv, \ \text{for all } t \geq 0 \text{ and } v \in (0, \varepsilon).$$

Then there exists an implementable RPI.

The feasibility of an RPI when capital is above the Golden Rule does not require knowledge of the elasticity of savings to interest rates–which was the main consideration of the analysis in the previous section for the constant-K policy. The key condition for the excess capital case is that aggregate consumption smoothly varies with transfers, ensuring that the increase in household expenditure can be financed with the increased net output.

4.2.2 Capital Below The Golden Rule

We now consider the case of $K^o < K^\star$. With a mark-up wedge between the return to capital and the interest rate, this case is consistent with either a positive or negative r^o.

The approach for generating a feasible RPI in this case is distinct from the over-accumulated capital case. In the latter, interest rates do not change, resources are generated from crowding out capital, and these resources are then rebated to consumers as lump-sum transfers. In Appendix A (Lemmas 4 and 5) we show that for the $K^o < K^\star$ case, increases in transfers alone are not a feasible path to generate an RPI, and they are not necessary either: when establishing feasibility, it is without loss to set transfers to their lowest possible level ($T_t = -(r_t - r^o)\underline{a}$).[25] Hence, we now focus on changes in interest rates.

Corollary 1 tells us then that a pair of sequences of interest rates, $r = \{r_t\}_{t=0}^{\infty}$, and transfers, $T = \{T_t\}_{t=0}^{\infty}$ is implementable if we can find an associated sequence of $\{K_t\}_{t=0}^{\infty}$ with $K_0 = K^o$ such

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[25]In Appendix A we require that aggregate consumption be weakly monotonic in transfers for these results.

that the aggregate consumption function satisfies

$$C_t(r, T) + K_{t+1} \leq F(K_t, N^o) + (1 - \delta)K_t.$$

To build towards the next result, recall that in the initial stationary economy, aggregate consumption is $C^o = F(K^o, N^o) - \delta K^o$. Letting $\widehat{C}_t \equiv C_t(r, T) - C^o$, Corollary 1 tells us that a fiscal plan is feasible if we can find a sequence $\{K_t\}$ such that for all t

$$\widehat{C}_t \leq F(K_t, N_0) - F(K_0, N_0) + K_t - K_{t+1} - \delta(K_t - K_0).$$

Define R_k to be the net marginal return to capital in the initial equilibrium:

$$R_k \equiv 1 + F_K(K^o, N^o) - \delta.$$

Given that K^o is strictly less than the Golden Rule, $R_k > 1$.

Suppose that production is linear in K, so $F(K_t, N^o) - F(K^o, N^o) + (1-\delta)(K_t - K^o) = R_k\widehat{K}_t$, where $\widehat{K}_t \equiv K_t - K^o$. We shall return to the general case of concave production below, but linearity allows us to build intuition towards the more general result.

Given a sequence $\{\widehat{C}_t\}$, the feasibility condition in (6) boils down to finding a sequence $\{\widehat{K}_t\}$ with $\widehat{K}_0 = 0$ and $\widehat{K}_t \geq -K^o$ (this latter guaranteeing that capital does not turn negative) such that the resource constraint holds:

$$\widehat{K}_{t+1} + \widehat{C}_t \leq R_k\widehat{K}_t, \tag{9}$$

for all $t \geq 0$. Solving forward and evaluating at $t = 0$,[26] a necessary condition for the consumption path $\{\widehat{C}_t\}$ to be feasible is

$$\sum_{t=0}^{\infty} R_k^{-t}\widehat{C}_t \leq 0. \tag{10}$$

That is, feasibility requires that the present value of consumption changes, *discounted at the marginal product of capital*, be less than the zero.

Condition (10) states the relevant inter-temporal price for assessing aggregate feasibility is the marginal product of capital, not the interest rate faced by households. If the government

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[26]Specifically, (9) implies

$$\widehat{K}_t \geq R_k^{-1} \sum_{s=0}^{T} R_K^{-s}\widehat{C}_{t+s} + R_K^{-T}\widehat{K}_T \geq R_k^{-1} \sum_{s=0}^{T} R_K^{-s}\widehat{C}_{t+s} - R_K^{-T}K_0.$$

Taking the limit as $T \to \infty$ and evaluating at $t = 0$ with $\widehat{K}_0 = 0$ gives us (10).

increases aggregate consumption in a period, this must be offset by a decrease somewhere else, where the increase and decrease are evaluated in present value terms using $R_k = 1 + F_K - \delta$. As we show below, condition (10) can be rewritten in terms of savings elasticities, connecting this result to our constant-K policy discussion.

However, before doing this, we extend this condition to the case of a general concave production function, marginal changes in interest rates, and obtain a sufficiency result. To do so, we need a strict inequality in the present value resource condition and a continuity condition. We first state the general result and then provide intuition for how we use these conditions:

Proposition 1. *Assume $\underline{a} = 0$, and $K^o < K^\star$. Consider a sequence $\widehat{r} = (0, r_1 - r^0, \dots)$, with $r_t \geq r^0$ for all $t \geq 1$ with at least one inequality strict. Suppose that there exist scalars $\varepsilon > 0$, and $h > 0$ such that*

$$\sum_{t=0}^{\infty} R_k^{-t} (C_t(r^o + \widehat{r}v, T^o) - C^o) \leq -hv, \ \text{for all } v \in (0, \varepsilon) \tag{i}$$

and there exists an $M > 0$ for which the following regularity condition holds

$$|C_t(r^o + \widehat{r}v, T^o) - C^o| \leq Mv, \ \text{for all } t \geq 0 \ \text{and } v \in (0, \varepsilon). \tag{ii}$$

Then there exists a feasible RPI.

As with Lemma 2, the result holds a particular "direction" of change fixed, and parameterizes distance in that direction by v, although in this case it is the interest rate sequence rather than transfers that changes. Condition (i) imposes a strictly negative upper bound on the present value of the changes in consumption. The strict inequality implies "extra" resources not used for consumption. The proof of the proposition uses this surplus to offset the second order implications of $F(K_t, N^o) - F(K^o, N^o)$ "missed" by the first-order term $R_k \widehat{K}_t$.

The Lipschitz continuity condition (ii) is used to ensure that we remain in the neighborhood of the initial equilibrium for a small change in interest rates at all times. This allows us to continuously govern the extent of changes in consumption and capital with the parameter v, placing an upper bound on the second order terms.

Recall that for the constant-K policy, aggregate consumption could not be higher than the initial consumption level in any period. With more general policies, consumption can deviate from the initial level in any direction. Proposition 1 says that the present value of these changes in consumption must be bounded above by zero. As with the constant-K policy, the key insight is that an increase in the interest rate must induce households to save in aggregate rather than increase consumption.

Note that the present value discount factor is the net return to physical capital, not the market interest rate. With $\mu > 1$, these will be different. Hence condition (i) of the proposition presents a simple and somewhat surprising separation between demand considerations (preferences) and supply (technology). The aggregate response of consumption to an interest rate change is determined by the initial distribution of wealth, household preferences, their idiosyncratic risk, and the interest rate that households face on their savings, which is captured by C_t. The role of technology is embedded in the discount factor used to sum over t. With a markup, the discount rate of the government to evaluate the feasibility of an RPI does not coincide with the market interest rate faced by households.

To gain more insight, let us narrow attention to just one interest rate change, say at time $\tau \geq 1$. That is, $\widehat{r} = \{0, 0, \ldots, \Delta r_\tau, 0, \ldots\}$, for $\Delta r_\tau > 0$. Note that $\partial C_t(r^o, T^o)/\partial r_\tau = dC_t(r^o + v\widehat{r}, T^o)/dv$ evaluated at $v = 0$ and $\Delta r_\tau = 1$. If this derivative exists and it is bounded in the neighborhood of $v = 0$ for all $t \geq 0$, then condition (ii) is satisfied, and for condition (i) it is sufficient that[27]

$$\sum_{t=0}^{\infty} R_k^{-t} \frac{\partial C_t}{\partial r_\tau} < 0. \tag{11}$$

If we can find such a τ, then we have an implementable RPI.

From Definition 4 and letting $R^o = 1 + r^o$, we have for $t \geq 1$

$$\frac{\partial C_t}{\partial r_\tau} = \begin{cases} R^o \frac{\partial \mathcal{A}_t}{\partial r_\tau} - \frac{\partial \mathcal{A}_{t+1}}{\partial r_\tau} \text{ for } t \neq \tau \\ R^o \frac{\partial \mathcal{A}_t}{\partial r_\tau} - \frac{\partial \mathcal{A}_{t+1}}{\partial r_\tau} + A^o \text{ for } t = \tau, \end{cases}$$

where $\partial \mathcal{A}_t/\partial r_\tau$ for $t \geq 1$ is defined in the same way as for aggregate consumption. As $\mathcal{A}_0 = A^o$ by definition, we have $\partial C_0/\partial r_\tau = -\partial \mathcal{A}_1/\partial r_\tau$.

Taking the discounted sum, (11) can be written

$$(R_k - R^o) \sum_{t=1}^{\infty} R_k^{-t} \frac{\partial \mathcal{A}_t}{\partial r_\tau} > R_k^{-\tau} A^o. \tag{12}$$

Define the elasticity of aggregate household savings at time t with respect to r_τ as

$$\xi_{t,\tau} \equiv \frac{\partial \mathcal{A}_t}{\partial r_\tau} \frac{R^o}{A^o}. \tag{13}$$

We can now state a corollary to Proposition 1:

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[27]To see this, let $G(v) \equiv \sum R_k^{-t}(C_t(r^o + v\widehat{r}, T^o) - C^o)$. Let $G'(0) = \lim_{v \to 0} G(v)/v$. Condition (11) says $G'(0) \leq -\hat{h} < 0$, for some $\hat{h} > 0$, which in turn implies that for $0 < \Delta < \hat{h}$ there is an $\varepsilon > 0$ such that for all $v \in (0, \varepsilon)$ we have $G(v)/v < -\hat{h} + \Delta \equiv -h < 0$, which is condition (i).

Corollary 2. *Assume $\underline{a} = 0$, and $K^o < K^\star$. Assume in addition that $\mathcal{A}_t$ is differentiable with respect to r_τ for some $\tau \geq 1$, and $\xi_{t,\tau}$ is defined by (13). If*

$$\left(\frac{R_k - R^o}{R^o}\right) \sum_{t=1}^{\infty} R_k^{-(t-\tau)} \xi_{t,\tau} > 1, \tag{14}$$

then an RPI is feasible.

This condition states that the present discounted value of savings elasticities, scaled by the gap between R_k and R^o, must be greater than one.

The fact that a large elasticity of savings is useful in making an RPI feasible is based on the same intuition as Figure 1, but now it is the present value elasticity and not just the steady-state elasticity. The sequence $\{\xi_{t,\tau}\}$ is related to the "sequence-space Jacobian" of Auclert et al. (2021), a point we discuss in the context of the calibrated model of Section 6.

The $R_k - R^o$ represents the difference in the inter-temporal price at which the government trades with "technology" versus at which it trades with households. This difference is governed by the markup. In particular, if the initial equilibrium has $\tau^k = 0$, then

$$R_k - R^o = F_k(K^o, N^o) - (r^o + \delta) = (\mu - 1)(r^o + \delta).$$

Thus, a larger markup aids in satisfying the feasibility condition.

We caution once more against concluding that a markup naturally implies a feasible RPI. An RPI is inconsistent with reducing pure profits or with providing subsidies to inputs financed with a lump-sum tax. The role of the markup here is that the feasibility condition recognizes that a government can transfer resources inter-temporally at rate R_k, while the households (in aggregate) choose not to do so due to market power.

We have shown that both a markup and a negative risk-free interest rate help make an RPI feasible. What if neither is present? That is, what if $F_K(K^o, N^o) - \delta = r^o > 0$. In this case, the government lacks the resources to implement an RPI:

Proposition 2. *Consider starting from a laissez-faire equilibrium with $\underline{a} = 0$, $\mu = 1$, and $K^o < K^\star$. Suppose that $\limsup_{t\to\infty} \mathcal{A}_t(r, T) = \infty$ implies $\limsup_{t\to\infty} C_t(r, T) = \infty$ for any non-negative sequence T. Then, there is no feasible RPI.*

Let us briefly comment on the main assumption for this result: it requires that household consumption be unbounded as household wealth increases without bound. This is a natural assump-

tion,[28] and is, for example, satisfied in the standard Aiygari environment.[29] Thus, the presence of $r^o < 0$ or a markup (or some combination of the two) is a necessary requirement for an RPI.

4.2.3 The Elasticity of Aggregate Savings

The existence of an RPI, at least locally to the initial equilibrium, depends on the weighted sum of aggregate savings elasticities given by (14). The $\xi_{t,\tau}$ are the impulse responses of aggregate savings $t - \tau$ periods after (or before, if negative) a one-time exogenous shock to the interest rate at τ. The key statistic is then a weighted sum of these responses, where the weight is given by the net marginal product of capital, and scaled by the difference between the marginal product of capital and the risk-free interest rate.[30] Conceivably, this statistic could be estimated using a vector autoregression, assuming one could identify a policy-induced change in the risk-free interest rate (or, equivalently, an exogenous change in government debt).

Testing the sensitivity of interest rates to changes in government debt or deficits was an active area of empirical research in the 1980s and 1990s.[31] Perhaps surprisingly, there are a number of empirical studies that conclude the Ricardian equivalence benchmark of no change in the interest rate, in the spirit of Barro (1974), is a reasonable description of the data. Nevertheless, there are other empirical estimates that conclude otherwise, and our reading of this literature is that there is no clear consensus.

In the Bewley-Huggett-Aiyagari literature, there are a few theoretical results. For example, for the case of CRRA utility, Benhabib, Bisin and Zhu (2015) show that as $a \to \infty$, the household saving function's sensitivity to the risk-free interest rate is increasing in the inter-temporal elasticity of substitution (IES). A similar result is proved by Achdou et al. (2021). Thus, the derivative with respect to r is governed by the IES, with a larger IES indicating a more elastic response, at least for the very wealthy. At the other end of the asset domain, Achdou et al. (2021) show that, for those at the lowest income realization and approaching the borrowing constraint, the sensitivity of savings to r also depends positively on the IES.

These results pertain to individual savings behavior at the extremes of the asset distribution. For a representative agent (RA) economy, this is enough. In the next section, we explore such an environment to gain some analytical insights. More generally, one needs to turn to computational examples, which we do in Section 6.

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[28]Recall that we have ruled out lump sum taxes, and hence a situation in which infinite private wealth is offset by an infinite household tax liability.

[29]For the argument, see Chamberlain and Wilson (2000), Lemma 2.

[30]Recall that if $R_k = R^o < 0$, then $F_K < \delta$ and we know an RPI is feasible from Lemma 2.

[31]See the surveys and associated references of Barth, Iden and Russek (1984); Bernheim (1987); Barro (1989); Elmendorf and Gregory Mankiw (1999); Gale and Orszag (2003); and Engen and Hubbard (2005).

5   A Representative Agent Economy

In this section, we use a Representative Agent (RA) economy to shade in some details behind the previous section's results. The RA economy's analytical tractability allows us to shed light on how government debt is used to "smooth" transfers, which may be necessary given that the short-run elasticity of aggregate savings will be smaller than the long-run elasticity. Moreover, the fact that an RPI is feasible in an RA economy establishes that the markup on its own, independent of risk sharing considerations or $r < 0$, opens the door to implementable RPIs, despite the fact that after-tax profits remain bounded below by the laissez-faire equilibrium and the government cannot resort to lump sum taxation.

5.1   The Aggregate Consumption Function

We assume the RA preferences are given by standard separable utility $\sum_{t=0}^{\infty} \beta^t u(c_t)$.[32] We assume that the economy starts from a laissez-faire equilibrium: $T^o = B^o = 0$. Note that in an RA economy with separable utility, in the steady state we have $r^o = 1/\beta - 1$.

For the RA economy, the aggregate consumption function $C_t(r, T)$ satisfies the Euler equation

$$u'(C_t) = \beta(1 + r_{t+1})u'(C_{t+1})$$

and the present value budget constraint

$$a_0 + \sum_{t=0}^{\infty} Q_t (Y^o + T_t - C_t) = 0, \tag{15}$$

where $a_0 = A^o = K^o$, $Q_t \equiv \left(\prod_{s=0}^{t}(1 + r_s)\right)^{-1}$, and $Y^o \equiv w^o N^o + \Pi^o$. We restrict attention to sequences of $\{T_t\}$ such that the present discounted value of transfers is bounded. The timing of the transfers does not matter for the household consumption allocation, only the discounted present value does.[33]

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[32] We assume that labor supply is exogenous and equal to N^o, which, as before, for our purposes will be equivalent to endogenous labor supply with zero wealth effect.

[33] As is well known, a RA may result from complete markets or Gorman aggregation. We do not specify the underlying household heterogeneity, but assume that no household is made worse off by an increase in the interest rate. In particular, $\underline{a} = 0$, which we assume does not bind for the exercises under consideration. This is obviously restrictive, but keep in mind we introduce the RA example for expositional reasons, not as a realistic description of the data. The model with uninsurable risk is obviously a more realistic model.

5.2 Transfer Smoothing: Implementing the First Best

It is instructive to explore whether the first best allocation constitute a feasible RPI, and whether government debt plays a role. Recall that the economy may be distorted by a markup, and the question we address in this subsection is whether, and how, the markup distortion could be removed without recourse to lump sum taxation. In what follows, the target allocation is the familiar efficient one from the neoclassical growth model absent the markup distortion, and thus we omit the derivation.

Starting from K^o, let $\{K_t^{FB}\}_{t\geq 0}$ denote the path of capital that would be chosen by a social planner with unlimited fiscal instruments to maximize the RA's welfare. Let $\{C_t^{FB}\}_{t\geq 0}$ denote the associated optimal consumption allocation. Let $\{R_t^{FB}\}_{t\geq 0}$ be the sequence of interest rates that decentralizes this consumption sequence, that is, $R_t^{FB} = (1 + r_t^{FB}) \equiv 1 + F_K(K_t^{FB}, N^o) - \delta$, $t \geq 1$, and with $R_0^{FB} = 1 + r^o$. We know that the the RA's Euler equation will be satisfied:

$$u'(C_t^{FB}) = \beta R_{t+1}^{FB} u'(C_{t+1}^{FB}).$$

As K^o is below the efficient steady state due to the markup distortion, and dynamics in the neoclassical growth model are monotone, we know that $\{K_t^{FB}\}$ is an increasing sequence. This implies that R_t^{FB} is decreasing over time. However, as $r^o = \lim_{t\to\infty} r_t^{FB} = 1/\beta - 1$, at every t the new interest rate sequence remains weakly greater than the initial interest rate – a requirement for an RPI. The question is how the government can implement this sequence without lump-sum taxation.

First, let us focus on the case without the use of government debt. In this first-best allocation, the government budget constraint must hold with equality, given that no resources are wasted. From Lemma 1, a binding government budget constraint implies that

$$T_t^{FB} = F(K_t^{FB}, N^0) - (r_t^{FB} + \delta)K_t^{FB} - \left(F(K^o, N^0) - (r^o + \delta)K^o\right).$$

for $t \geq 1$ and $T_0^{FB} = 0$. As $\{K_t^{FB}\}$ is an increasing sequence and F is concave, the sequence of transfers $\{T_t^{FB}\}$ is increasing over time for $t \geq 1$. Hence, if $T_1^{FB} \geq 0$, the sequence $\{r_t^{FB}, T_t^{FB}\}$ constitutes a feasible RPI.

However, if $T_1^{FB} < 0$, it still may be possible to implement the first best using government bonds. In particular, the first best can be implemented as an RPI as long as

$$0 \leq \sum_{t=0}^{\infty} Q_t T_t^{FB} = \sum_{t=0}^{\infty} Q_t \left[F(K_t^{FB}, N^0) - F_K(K_t^{FB}, N^o)K_t^{FB} - \left(F(K^o, N^0) - (r^o + \delta)K^o\right)\right].$$

If this inequality holds, but $T_t^{FB} < 0$ for some interval $t \leq \bar{t}$, the government can issue bonds

during the initial periods to cover the shortfall and avoid negative transfers. As long as the present value is weakly positive, the government budget constraint will be satisfied. This is what we mean by the role of government debt in "transfer smoothing." Note that there is no guarantee that the inequality will hold. This highlights that the first best may not be attainable without resorting to lump-sum taxation, even with the ability to smooth transfers using debt. In Appendix B we provide numerical results that highlight the role of the intertemporal elasticity substitution in making the efficient path an RPI.

The takeaway from this exercise is that to implement the first best, the government must subsidize investment to build up the capital stock. This requires a higher interest rate for households and a subsidy to firms. The short-run elasticity of aggregate savings is smaller than the long-run elasticity (which is infinite with separable utility), requiring an overshooting of the interest rate in the short-run relative to the steady state. The government can smooth this cost by using government bonds. In this manner, capital investment and government debt are *complements* rather than substitutes along the transition.

5.3 The Inter-temporal Elasticity of Substituion

One advantage of the RA example is the close link between the elasticities of aggregate savings appearing in Corollary 2 and the inter-temporal elasticity of substitution (IES) of the representative consumer. Consider the same perturbation used in the corollary; namely, a single period τ in which $r_\tau > r^o$, with every other period setting $r_t = r^o$. As $r^o = 1/\beta - 1$ in the RA economy, consumption is constant before and after τ, with a one-time increase between $\tau - 1$ and τ. In particular,

$$C_t(\boldsymbol{r}, \boldsymbol{T}^o) = \begin{cases} \underline{c} & \text{for } t \leq \tau - 1 \\ \overline{c} & \text{for } t \geq \tau, \end{cases}$$

where $\underline{c}$ and $\overline{c}$ solve the Euler equation and the present value budget constraint:

$$u'(\underline{c}) = \beta(1 + r_\tau)u'(\overline{c})$$

$$0 = R^o A_0 + \frac{1 - (R^o)^{-\tau}}{1 - 1/R^o}(Y^o - \underline{c}) + \frac{1}{(R^o)^{\tau-1}R_\tau(1 - 1/R^o)}(Y^o - \overline{c}),$$

where again we use $Y^o = w^o N^o + \Pi^o$.

As in the main analysis, the key behavioral response is whether the private household is willing to postpone consumption due to the increase in interest rate. In the RA case, this is governed by the IES. In the appendix we prove that, for the representative agent case, our sufficient condition in Corollary 2 holds for some t if the IES is large enough:

Lemma 3. *Let $\zeta \equiv -u'(C^o)/(u''(C^o)C^o)$ denote the IES evaluated at the initial consumption level. If $\mu > 1$ and*

$$\zeta > \frac{r^o A^o}{C^o},$$

then there exists an implementable RPI.

The necessity of $\mu > 1$ follows from the RA assumption, as the markup represents the only inefficiency that can be potentially corrected. The role of the IES reflects that a more elastic response to a change in interest rate makes an RPI easier to implement. The term on the right-hand side of the inequality reflects the wealth effect of higher interest rates. In particular, it is the fraction of initial consumption financed with asset income, or one minus the share of net income paid to labor and profits. The larger the asset share becomes, the more the interest rate increase induces the consumer to raise consumption. Note that this ratio is strictly less than one, and hence an IES greater than one is sufficient to satisfy the condition.

The takeaway from this exercise is to confirm our focus on the elasticity of aggregate savings (and consumption) to a change in the interest rate as the gatekeeper to a feasible RPI. In the RA economy, this boils down to the tradeoff between the IES and the share of income paid to capital. In the heterogeneous household model, we cannot map the sequence of elasticities to a single preference parameter. For that model, we turn to calibrated simulations.

6   Simulations

In this section, we present simulation results for various policy experiments. Our benchmark focuses on the dynamically efficient economy with markups. In this setting, we explore constant-K policies as well as policies with capital crowding in. For contrast, we also briefly present a competitive economy that is dynamically inefficient with capital crowding out.

The simulated economies allow us to assess the scope for Pareto-improving fiscal policies in a calibrated quantitative model and to compute transition dynamics and welfare implications. The quantitative experiments will also underscore how government debt is used in implementing Pareto-improving policies.

6.1   Parameter Settings

The utility function we consider for households is of the Epstein-Zin form

$$V_{it} = \left\{ (1-\beta)x_{it}^{1-1/\zeta} + \beta \left(\mathbb{E}_z V_{it+1}^{1-\gamma} \right)^{\frac{1-1/\zeta}{1-\gamma}} \right\}^{\frac{1}{1-1/\zeta}},$$

where β is the discount factor, ζ is the elasticity of inter-temporal substitution, γ is the risk aversion coefficient, and x is the composite of consumption and labor:

$$x_{it} = c_{it} - n_{it}^{1/\nu}.$$

The parameter ν controls the Frisch elasticity of the labor supply. We set some of the preference parameters to conventional values in the literature and others as part of the calibration. The elasticities of substitution and of labor supply are set to the common parameter values of 1 and 0.2, respectively. The discount factor and coefficient of risk aversion are set as part of the calibration exercise described below. We set the borrowing constraint to zero for all households.

An important part of the parametrization is the stochastic structure for idiosyncratic shocks. We adopt the structure and estimates from Krueger, Mitman and Perri (2016), which use micro data on after-tax labor earnings from the PSID. Idiosyncratic productivity shocks z_{it} contain a persistent and a transitory component, and their process is as follows:

$$\log z_{it} = \tilde{z}_{it} + \varepsilon_{it}$$
$$\tilde{z}_{it} = \rho^z \tilde{z}_{it-1} + \eta_{it},$$

with persistence ρ^z and innovations of the persistent and transitory shocks (ε, η), and associated variances given by $(\sigma_\varepsilon^2, \sigma_\eta^2)$. We set the three parameters controlling this process $(\rho^z, \sigma_\varepsilon^2, \sigma_\eta^2)$ to .9695, .0384, and .0522, respectively, to reflect the estimated earnings risk in Krueger, Mitman and Perri (2016) for employed individuals. We discretize this process into 10 points, based on Tauchen (1986).

We take a parsimonious approach to allocating profits to households. In particular, we assume a distinct class of entrepreneurs who are endowed with managerial talent and consume profit distributions in a hand-to-mouth manner. While stark, this approach offers a number of advantages. First, it approximates that a significant share of entrepreneurial rents accrues to a small portion of the population. Second, under this assumption, profits do not affect factor prices, and so we can solve the economy without taking a stand on the idiosyncratic details of the entrepreneurial class. Finally, and related to the previous point, the analysis is invariant to the extent to which profits are offset by fixed costs versus representing pure rents.

The technology specification is Cobb-Douglas, $F(K, N) = K^\alpha N^{1-\alpha}$. We use standard values for the coefficient α and for the depreciation rate of capital δ. The values are $\alpha = 0.3$ and $\delta = 0.1$. The markup parameter μ is set to 1.4, which is within the range of estimates in Basu (2019).[34]

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[34] As noted above, some of this markup may represent fixed costs. The aggregate markup may also reflect smaller markups at different stages of production in a vertical supply chain, as in Ball and Mankiw (2021). In fact, 1.4 is close to the number they use in their numerical exercises.

We calibrate the discount factor and the coefficient of relative risk aversion by targeting a capital-output ratio of 2.5, based on Aiyagari and McGrattan (1998) and Krueger, Mitman and Perri (2016), and an interest rate of -1.4%, which is the difference between the average one-year treasury rate and average nominal GDP growth in the United States since 1962.[35] While our focus is on Pareto-improving policies relative to a laissez-faire benchmark, the empirical moments are generated from an economy with government debt. Hence, we simulate a stationary economy with a debt-to-output ratio of 60%, which is the average value in the U.S. since 1966, and choose preference parameters to match moments from this economy to the data. The resulting values are $\{\beta = 0.993, \theta = 5.5\}$.

We treat the economy with debt as being generated from a Pareto-improving fiscal policy starting from the laissez-faire economy. That is, in simulating the economy with government debt during moment matching, we assume that tax policy is such that after-tax wages and capital are the same as in the laissez-faire economy. We refer to this constant-K policy as our "baseline fiscal policy".

Table 1: Baseline Constant-K Policy and Laissez-Faire Economies

	Data	Constant-K Policy	Laissez-Faire
Aggregates			
Public Debt (% output)	60	60	0
Interest Rates(%)	-1.4	-1.4	-1.7
Capital (rel. output)	2.5	2.6	2.6
Wealth Distribution			
Q1 Wealth Share	-1	1	1
Q2 Wealth Share	1	4	4
Q3 Wealth Share	4	11	10
Q4 Wealth Share	13	24	23
Q5 Wealth Share	83	61	63

Table 1 presents some moments in the steady states of the economy with baseline constant-K fiscal policy and the laissez-faire economy. The levels of public debt, interest rates, and capital in the economy with the baseline fiscal policy match the data moments by construction.[36] The table shows that an increase in debt to output of 60% raises interest rates by 0.3%. We also present some moments on the wealth distribution in the steady states—namely the wealth share of each asset quintile—and compare them with data as reported in Krueger, Mitman and Perri (2016). Our model economies generate skewed distributions of wealth, with most of the wealth being held by

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[35]This estimate is consistent with the ones in Blanchard (2019) and Mehrotra and Sergeyev (2020).

[36]The economy is dynamically efficient, also by construction. To see this, $F_K = \alpha Y/K = 0.3/2.5 = 0.12$, which is greater than the depreciation rate of 0.10.

the top quintile of the distribution, although they are not quite as skewed as the data. In addition in our model economies, a small fraction of agents, about 2%, are at their borrowing constraint at any period.

6.2 Robust Pareto Improvements

We now describe the transition as the government implements its fiscal policy and assess the scope for Pareto improvements. The economy starts in the laissez-faire steady state and transitions to the steady state with fiscal policy. We perform two policy experiments, one in which capital is held constant, the constant-K policy, and one in which capital eventually reaches the Golden Rule level, the crowding in policy. Both of these experiment are undertaken in the an economy with $F_k > \delta$. We also perform sensitivity over preference parameters.

The two experiments confirm that Pareto-improving fiscal policies are feasible in a calibrated model. In particular, the calibrated aggregate savings schedule is sufficiently elastic to allow the government to increase both debt and capital without resorting to lump-sum taxation.

6.2.1 Baseline Constant-K Policy

We start with a policy plan that transitions from a laissez-faire stationary equilibrium to a new steady state with the baseline fiscal policy. In particular, the government takes the economy from zero debt to a level of 60% of output, while keeping after-tax wages and profits constant. Our posited path of public debt is depicted at the top left panel of Figure 2; debt increases monotonically until it reaches its steady-state level of 60% of output. Also by construction in the constant-K policy, capital is held fixed at the laissez-faire level, as depicted in the top middle panel of Figure 2. Given the policy of constant capital and wages, output and consumption (reported in the lower middle panel) do not change. This will be different in the subsequent experiment with capital crowding in.

Given this path of debt and capital, we solve for the equilibrium interest rates path r_t and associated government transfers T_t. The computational algorithm and other details are reported in Appendix D.

The top right panel of Figure 2 plots the path for government transfers and seigniorage revenue from debt issuance $B_{t+1} - (1 + r_t)B_t$, both relative to output. Transfers are larger on impact—about 5% of output—remain positive throughout the transition, and settle to a small positive level of about 0.1% of output in the steady state. The difference between transfers and seigniorage revenue is equal to the tax revenues, which are negative owing to the capital subsidies.

The bottom left panel in Figure 2 plots the path for the interest rate. Interest rates rise with public debt to induce households to hold a greater stock of aggregate wealth. Note that interest

rates overshoot during the transition. This outcome shows Le Chatelier principle at work; the short-run elasticity of assets to interest rates is lower than its long-run level. The sharp spike in interest rates makes the policy fiscally expensive, as exposited in Section 3. However, the cost is more than offset by the funds raised directly by debt issuance, as evinced by the elevated transfers early in the transition.

The bottom right panel plots the dispersion of household consumption relative to the laissez-faire dispersion. Consumption dispersion decreases by about 10% upon the introduction of the fiscal policy plan, as households with low assets and low productivity benefit from government transfers that support their consumption. As transfers fall over time, consumption dispersion increases but remains about 2% below the one in the laissez-faire economy. The smaller long-run consumption dispersion reflects that households on average hold a greater stock of precautionary savings, given the elevated interest rate.
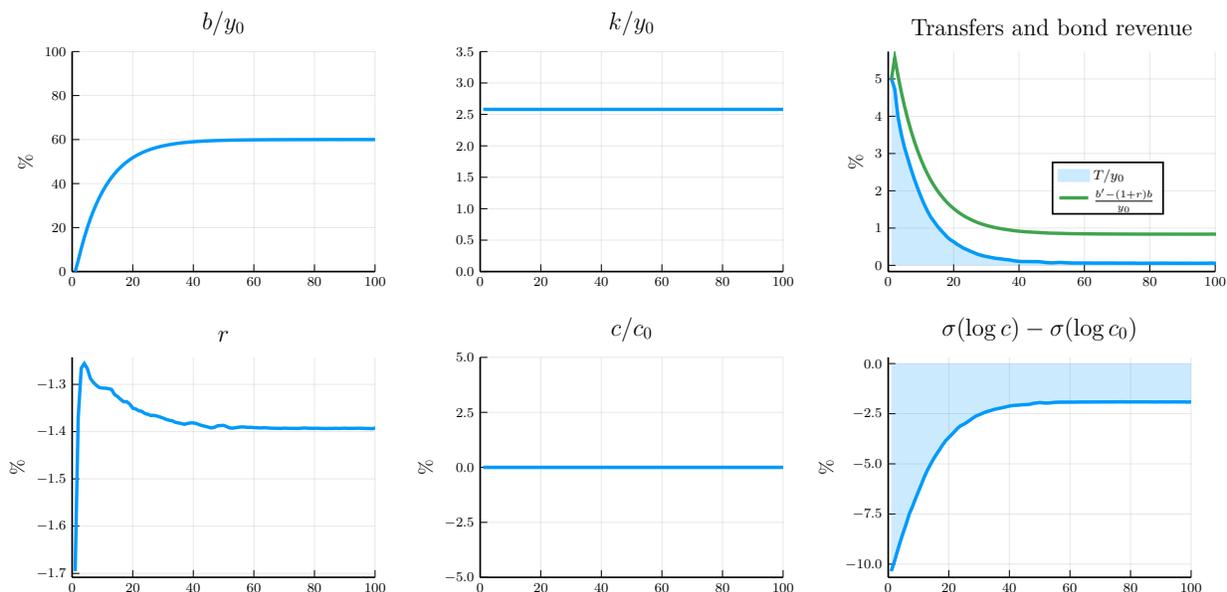

Figure 2: Constant-K Policy Transition

The transition paths of positive transfers and higher interest rates imply that our baseline constant-K fiscal policy is Pareto improving. We now evaluate the magnitude of the welfare gains. Table 2, Column 1 reports welfare for various households upon the announcement of the policy. Welfare is measured in consumption equivalence units relative to the laissez-faire economy. Across the distribution of households for assets and productivity (a, z), the economy with fiscal policy delivers higher welfare for every household. The table reports five measures of welfare gains: the mean gain; the minimum gain; and the mean gains for the bottom 10%, the 40-60th percentiles, and the top 10% of the asset distribution. The mean welfare gains are computed by integrating over idiosyncratic states, conditional on belonging to the respective

asset bin, weighted by the invariant distribution of the laissez-faire economy.

The mean welfare gain is 2.62%, and the minimum gain is 2.09%. Looking across the wealth distribution, welfare gains are greatest for the poorest households. While all households receive the same transfer, the poorer households benefit relatively more in percentage terms. However, gains are not monotonic in wealth. The top decile of asset holders experience a greater welfare gain than those in the middle of the asset distribution. This reflects the fact that the benefits of a higher interest rate are increasing in wealth. At some point in the distribution, this effect dominates the uniform transfer, generating a non-monotonicity in percentage welfare gain as a function of initial wealth.

We can also compute welfare gains comparing the new steady state to the laissez-faire steady state, ignoring the transition. Because of the declining path of transfers, welfare gains are more modest in the new steady state relative to the gains enjoyed at $t = 0$. Nevertheless, all households are better off in the new steady state, with an average welfare gain of 1.8%.

Table 2: Changes in Welfare

	Constant-K Policy	Crowding In Policy
Welfare Gains at Announcement (%)		
Overall Mean	2.62	5.16
Overall Minimum	2.09	4.48
Poor ($\leq$ 10 pct)	3.57	5.19
Middle Wealth (40-60 pct)	2.30	4.95
Rich (>90 pct)	2.82	7.12

The preceding analysis established that a Pareto-improving fiscal policy targeting a long-run debt-to-output level of 60% is feasible. This result reflects the fact that a debt level of this magnitude can be absorbed by households, with only a modest increase in interest rates. This raises the question of whether even higher levels of debt are feasible while still ensuring all agents are weakly better off. To answer this question, we revisit the logic of Figure 1. In particular, long-run seigniorage is given by $-rB$, while the costs are captured by $\Delta r \times K_0$. In Figure 3, we plot these two components for stationary equilibria with different levels of debt to output for the constant-K policy. At each debt level, tax policy is set to deliver laissez-faire wages and profits.

Up until debt levels of roughly twice the level of output, seigniorage exceeds fiscal costs, implying positive lump-sum transfers to households. Beyond this level of debt, the increase in interest rates makes weakly positive transfers infeasible. Note that these two curves intersect while seigniorage is still increasing in debt. Eventually, r becomes close enough to zero that seigniorage begins to decline in debt. The peak of this Laffer curve occurs at debt levels roughly

four times output. Feasible Pareto-improving levels of debt, however, are much lower than this peak.[37]

While Figure 3 establishes only that the policy is feasible in the new steady state, the analysis of transition dynamics in the baseline case above suggests that feasibility in the steady state is the critical metric. Along the transition, the government is a net issuer of bonds. As long as the revenue from the net issuances dominates any overshooting of the interest rate, feasibility rests on long-run considerations.
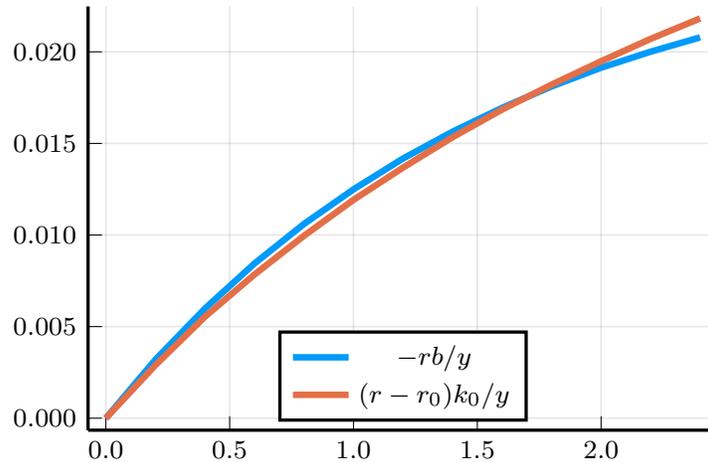

Figure 3: Steady-State Seigniorage and Tax Revenue across Debt

6.2.2 The Role of Preferences

Section 3 revealed that a key consideration in the feasibility of Pareto-improving policies is the elasticity of the aggregate savings function with respect to the interest rate. The short-run and long-run elasticities, in turn, depend on household preferences and the extent of exposure to idiosyncratic risk in a non-trivial manner. To assess this sensitivity quantitatively, we perform comparative static exercises to our baseline constant-K policy experiment by varying household preferences. We vary the inter-temporal elasticity of substitution (IES), the coefficient of risk aversion, and the discount factor (β), and we trace out the path of interest rates and transfers in response to an increase in government debt. We present the case for alternative IES parameters and address the sensitivity to risk aversion and β in the Appendix. In all experiments, the increase in government debt is as in the baseline reaching 60 percent of initial aggregate output; keep in mind that the laissez-faire capital stock varies across experiments.

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[37]Bassetto and Sargent (2020) argue in an OLG framework that the peak of the debt Laffer curve may occur while r is strictly below the growth rate, making $r < g$ an unreliable guide for expanding government funds via debt issuance.

In Figure 4, we plot the path of interest rates (panel (a)) and transfers (panel (b)) for different values of the IES—namely, 0.5, the benchmark 1.0, and 1.5, holding all other parameters as in the benchmark, including risk aversion. The pattern confirms the conjecture that a higher IES requires a smaller increase in interest rates to absorb the government debt, in both the short-run and long-run.

The value of the IES, however, also affects the initial level of the interest rate. In particular, a low elasticity implies lower level of interest rates throughout the transition, which also matters for the feasibility of Pareto improving policies. As seen in the path for transfers in Figure 4, it is the *more* elastic preferences that eventually require negative transfers (albeit very small ones, on the order of 10^{-4} of initial output). With higher elasticity, while the small increase in interest rates requires a small subsidy to capital, the fact that rates are close to zero implies lower seigniorage revenue, and the latter slightly dominates the former and impedes a Pareto improvement.[38]

Appendix Figures A.2 and A.3 contain the same simulated time series for alternative coefficients of relative risk aversion (CRRA) and time discount factors β. Intuitively, a higher CRRA implies a lower laissez-faire interest rate as well as a less elastic aggregate savings function. Nevertheless, the former dominates, implying feasibility for a CRRA of 10.0 but not for a CRRA of 2.0. Similarly, a lower discount factor (more impatience) implies a higher initial interest rate, leaving less fiscal space for Pareto-improving policies.

Figure 4: Alternative Inter-temporal Elasticities
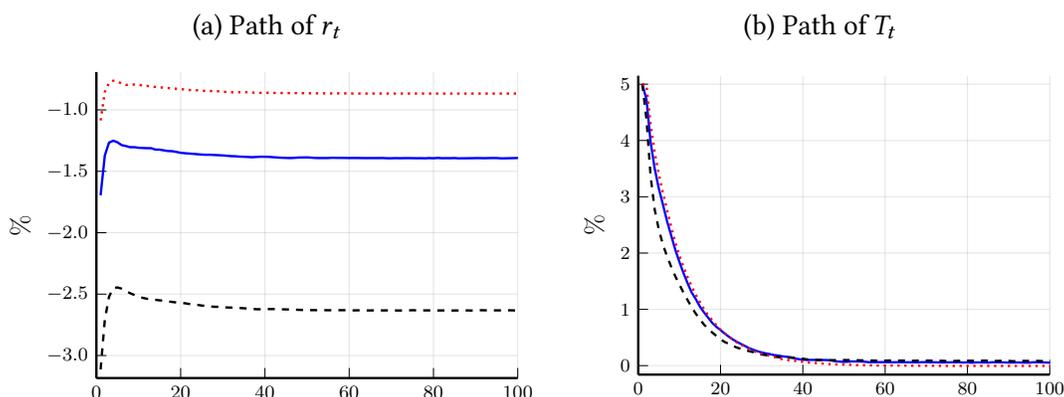
(a) Path of r_t · (b) Path of T_t

Note: This figure displays the path of interest rates (Panel (a)) and transfers as a percentage of laissez-faire output (Panel (b)) associated with the baseline fiscal policy under alternative preference parameterizations. In both panels, the solid line is the benchmark IES=1.0; the dotted red line is IES=1.5; and the dashed black line is IES=0.5.

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[38]The fact that the laissez-faire interest rate varies with the IES while holding risk aversion constant stems from the fact that precautionary savings depend on more than the extent of risk aversion. Kimball and Weil (1992) show that with Kreps-Porteus preferences, the strength of the precautionary savings motive is determined by attitudes towards both risk and inter-temporal substitution.

6.2.3 Crowding In Policy

We now consider a fiscal policy plan that engineers an increase in capital that reaches the Golden Rule level in the new steady state. In particular, with this crowding in policy, capital relative to output increases from 2.5 to 3.0.[39] We assume that the government also pursues the same path of debt issuance as in the previous experiment. Capital subsidies are set to ensure firms rent the targeted level of capital given the prevailing interest rate, and labor and profit taxes are set so that after-tax wages and profits remain constant. We find that transfers are positive throughout, and hence the fiscal plan is a feasible Pareto improvement.
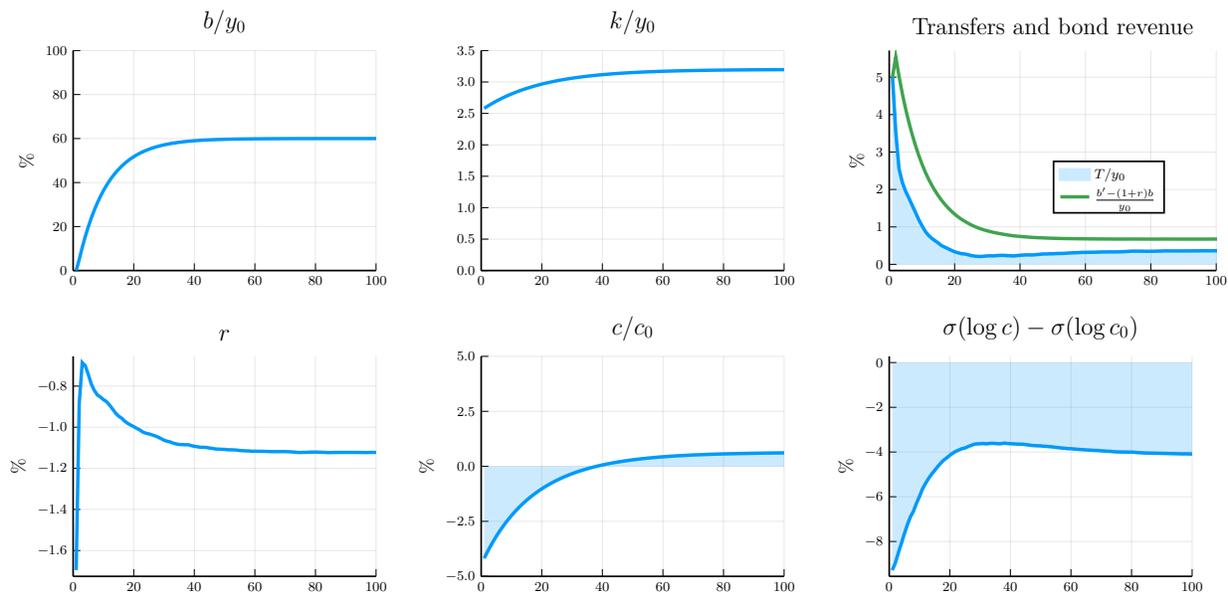

Figure 5: Crowding In Policy Transition

Figure 5 plots the variables of interest during this transition. The layout of the figure is the same as that of Figure 2. Along the path, we normalize quantities by the initial laissez-faire income, keeping in mind that contemporaneous income is increasing with capital.

The first two panels of the figure's top row present the posited paths of debt and capital. The top right panel illustrates that government transfers are positive throughout the transition. They fall in the middle of the transition and increase towards the end. Transfers increase towards the end because interest rates are declining and capital is increasing, easing the fiscal burden.

As in the constant-K policy experiment, seigniorage revenue from borrowing falls during the transition but settles at a lower level, owing to the higher interest rates. As seen in the bottom left panel of the figure, interest rates rise more with a fiscal policy that crowds in capital, because households need to be induced to hold the additional capital as well as debt.

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[39]Recall that $F_K = \alpha Y/K$ and $\delta = 0.10$, and hence given $\alpha = 0.3$, the Golden Rule is achieved at $K/Y = 3.0$.

The bottom middle panel shows that aggregate consumption falls early in the transition, as the economy increases investment in new capital, and settles above the laissez-faire level in the new steady state with higher capital. However, throughout the transition, the dispersion of household consumption remains uniformly below the level in the laissez-faire economy. As seen in the bottom right panel, the standard deviation drops about 9%, and increases to about 4% lower than in the laissez-faire. Thus, fiscal policy improves risk sharing because of larger transfers as well as a larger stock of household wealth, which earns a higher rate of return.

The second column of Table 2 reports the welfare gains for this experiment. Welfare increases for all households both upon the fiscal policy announcement and also in the new steady state. The mean welfare gain is 5.16%, and the minimum gain is 4.48%. In this case, fiscal policies benefit the rich households more than poor households, with gains upon impact of 7.12% and 5.19%, respectively. Nevertheless, as before, households in the middle of the wealth distribution gain the least in percentage terms. The gains in this policy experiment are much larger than for the baseline policy, because they reflect not only better risk sharing but also a higher level of capital and consumption in the long run.

The crowding in experiment assumed that the government issued debt in the same manner as it did in our baseline policy. In the analysis of Section 3, we argued that debt issuance may be useful along the transition to a higher capital stock, if the short-run elasticity of household savings is significantly lower than the long-run elasticity. This configuration made debt issuance a complement to capital accumulation. We can use the quantitative model to explore this property in greater depth.

Specifically, we study an alternative fiscal policy that implements the same path of capital as in our crowding in experiment, but with zero debt issuance. The transition dynamics for this case are presented in Figure 6. The top right panel of the figure shows that without public debt, the government needs to lump-sum tax households early in the transition. The large increase in the interest rate necessary to induce households to hold more wealth (the bottom left panel) implies large fiscal costs from capital rental subsidies. In the transition with debt, the government could use debt issuance to smooth this burden. Without debt, the government must lump-sum tax early in the transition, implying some households may be strictly worse off. These losses are also reflected by the higher standard deviation of consumption early in the transition, which is plotted in the lower right panel. Increasing capital towards a more efficient level without debt is not a feasible Pareto improving policy, because it requires lump-sum taxes during the transition, despite delivering positive transfers in the new steady state.

This experiment suggests that public debt is an important tool in Pareto-improving capital expansions. In this sense, government debt and capital expansions can be complements, rather than substitutes, as they have been traditionally regarded, providing a contrast with Diamond
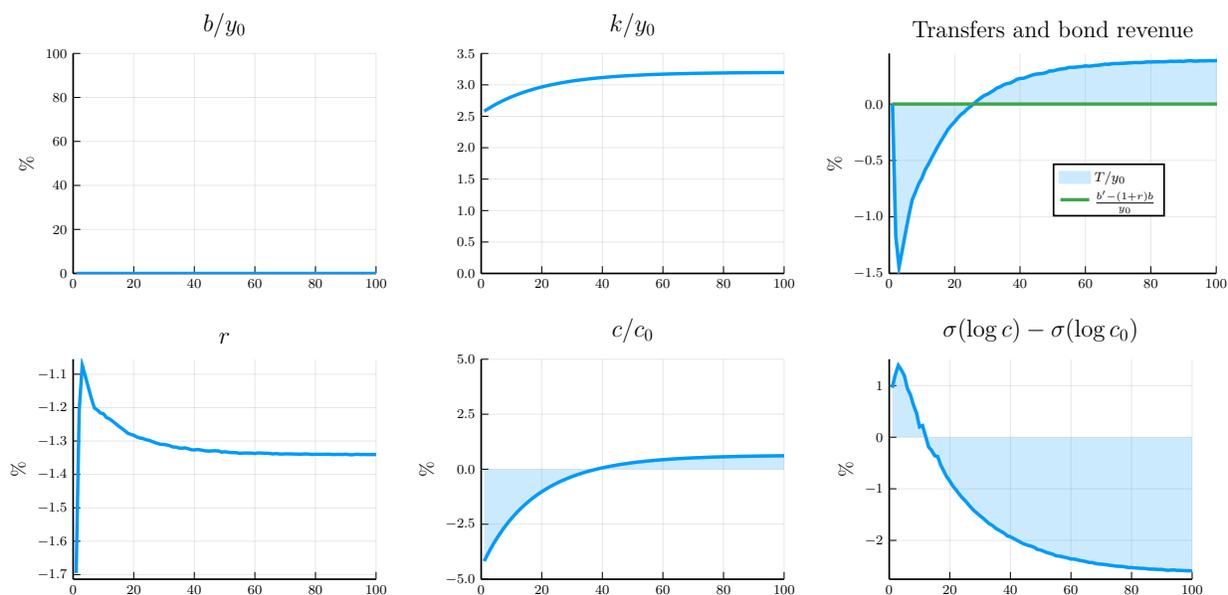
(1965).

Figure 6: Fiscal Policy Transition with Crowding In and No Debt

7 Conclusion

We provided sufficient conditions for the feasibility of Pareto-improving fiscal policies in the Bewley-Huggett-Aiyagari model when the risk-free interest rate on government bonds is below the growth rate ($r < g$) or there is a markup ($\mu > 1$). The key condition is that seigniorage revenue raised by government bonds exceeds the increase in the interest rate times the initial capital stock. As long as the aggregate household savings schedule is sufficiently elastic and/or the markup is large, such Pareto-improving policies are feasible. In this sense, we have shown that the feasibility of a Pareto improvement depends on an aggregate elasticity, not on the finer details of idiosyncratic heterogeneity that underpin this elasticity. In calibrated examples using U.S. data on household heterogeneity and historical data on interest rates and growth rates, we find scope for Pareto improving policies for a wide range of debt and tax policies.

The government uses seigniorage debt revenue to provide transfers to households and to subsidize factor prices. These policies are welfare improving for all households because they improve risk sharing and can give rise to beneficial supply expansions. We find scope for Pareto improving fiscal policies with and without capital crowding in and in both dynamically efficient and inefficient economies. We find that debt is a useful tool, especially for fiscal policies that expand capital.

Many governments around the world are rapidly expanding their public debt in the context of low interest rates. Our analysis points to a force that increases the benefits of such expansions. The analysis provided simple conditions for fiscal feasibility, complementing the typical dynamic inefficiency condition of Samuelson (1958) and Diamond (1965). We have tried for analytical clarity in an extension of the canonical Bewley environment, rather than a full fledged quantitative model for policy design in the current context. In particular, we have abstracted from aggregate risk. Integrating the possibility of rising interest rates in response to aggregate shocks would certainly increase the fiscal costs of higher debt. The benefits from increasing debt to improve risk sharing and for supply expansions would then have to be balanced against the costs of having to tax future generations to pay for the debt if interest rates rise.

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Appendix A   Transfers When Capital is Below the Golden Rule

Consider the following notion of monotonicity of aggregate consumption with respect to transfers:

Definition 5. *We say $C = \{C_t\}_{t \geq 0}$ is weakly increasing in T if $T' \geq T$ implies $C(r, T') \geq C(r, T)$, where the inequality holds for all t in the respective sequences. If $T' \geq T$ for which there is a t such that $T'_t > T_t$ implies $C(r, T') \geq C(r, T)$ and that there is an s such that $C_s(r, T') > C_s(r, T)$, we say C is strictly increasing in T.*

This is a natural property, in that holding constant all interest rates, one would naturally expect an increase in lump sum transfers would induce households (in aggregate) to consume more.[40]

The following result says that if consumption is weakly increasing in transfers, then we can ignore the role of transfers when looking for an RPI (as long as an interest rate have changed). That is, transfers are not *necessary* for implementability:

Lemma 4 (Transfers are not necessary). *Suppose that C is weakly increasing in T. Let (r, T) be an implementable RPI where for some t, $r_t > r^o$. Then (r, T') where $T' = \{-(r_t - r^o)\underline{a}\}_{t \geq 0}$ is also an implementable RPI.*

Proof. Note that an RPI requires that $r_t \geq r^o$ and $T_t \geq -(r_t - r^o)\underline{a}$. The fac that the C is weakly increasing implies that $C_t(r, T) \geq C_t(r, T')$, as $T \geq T'$. The sequence of K_t that implements the (r, T) then also implements (r, T'). Given that $r_t > r^o$ for some t, it follows that (r, T') is an implementable RPI. □

The following result says that if consumption is strictly increasing in transfers, than an RPI is not implementable without a change in an the interest rate. That is, transfers alone are not *sufficient*:

Lemma 5 (Transfer are not sufficient). *Suppose that C is strictly increasing in T. If $K^o < K^\star$, then there is no implementable RPI in which $r = r^o$.*

Proof. Suppose there is an implementable RPI, (r^o, T). There must be a non-negative sequence of $\{K_t\}_{t=0}^{\infty}$ such that

$$C_t(r^o, T') + K_{t+1} \leq F(K_t, N^o) + (1 - \delta)K_t$$

Exploiting the concavity of technology, and that $C^o = F(K^o, N^o) - \delta K^o$, we have that

$$K_{t+1} - K^o \leq (R_k - 1)(K_t - K^o) - (C_t(r^o, T) - C^o).$$

Note that $R_k > 1$, together with C increasing in T, implies that $K_t \leq K^o$ for all t.

---

[40]For an individual agent in incomplete markets it is possible to construct examples where individual consumption falls given an increase in future transfers. However, we are counting on heterogeneity to guarantee that such individual behavior does not aggregate. Wolf (2021) presents examples of permanent income and hand to mouth households where these assumptions hold. See also Farhi, Olivi and Werning (2022) for general comparative statics results for incomplete market economies.

Let s be the first time where $C_{s-1}(r^o, T) > C^o$ (such a time exists, given that C is strict increasing in T). Then, the above implies that $K_s < K^o$. Now note that

$$K_{s+m} - K^o \leq (R_k - 1)^m(K_s - K^o)$$

Given that $R_k > 1$, it follows then that $K_t < 0$ for t large enough, a contradiction of implementability. □

When we focus on the case where the economy operates below the Golden Rule, the above result tells us that in an implementable RPI (under a reasonable assumption on C) an interest rate must changes at some date. The reason is that with only increases in transfers, aggregate consumption will be higher at all times with the RPI than originally, an impossibility given the resource constraint and $K^o < K^\star$.

Appendix B Numerical Results for the Representative Agent

Figure A.1 shows the parameter region of the inter-temporal elasticity of substitution and markups μ such that there neoclassical efficient path constitute an RPI (for a given discount factor and other technology parameters). As can be seen, a higher elasticity of intertemporal substitution or a higher markup facilitates the implementability of the RPI. With a higher elasticity of substitution, households are willing to save larger amounts for small increases in interest rates, generating faster convergence. Smaller increases in interest rates and faster convergence increase the present discounted value of T^{FB}. With higher markups, initial capital K^o is more depressed relative to the efficient one, and this difference also tends to increase the present discounted values of transfers.
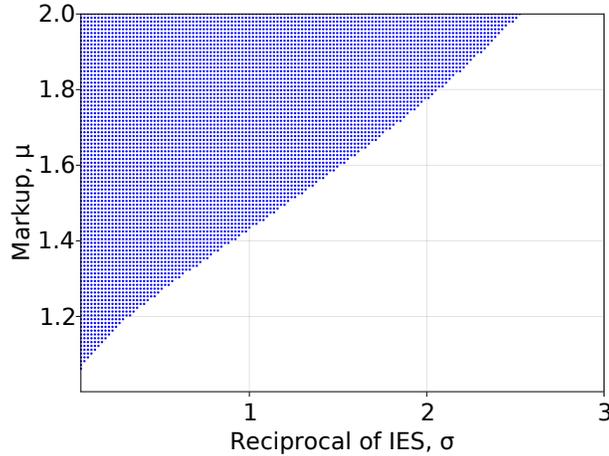

Figure A.1: A neoclassical example. Letting $u(c) = c^{1-\sigma}/(1 - \sigma)$, the shaded area represents values of μ and σ where the neoclassical solution is an RPI. In this case σ is the reciprocal of the intertemporal elasticity of substitution (IES). For this computation, the rest of parameters are $\beta = 0.95$, $F(K, 1) = K^\alpha$, with $\alpha = 0.33$, and $\delta = 0.1$.

Appendix C   Proofs

C.1   Proof of Lemma 1

For $t \geq 0$, set τ_t^n such that

$$\frac{F_N(K_t, N_0)}{(1 + \tau_t^n)\mu} = w_0.$$

This ensures the labor market clears at $w_t = w_0$ and $N_t = N_0$, where GHH preferences ensure that the households are willing to supply N_0 at wage w_0. Note that as K_0 is given, τ_0^n is the same as the initial equilibrium. Similarly, the government taxes or subsidizes profits so that

$$\Pi_t = (1 - \tau_t^\pi)\widetilde{\Pi}_t = (1 - \tau_t^\pi)(\mu - 1)F(K_t, N_0)/\mu = \Pi_0.$$

Finally, the government must ensure that the representative firm's choice of capital is consistent with the risk-free interest rate:

$$F_K(K_t, N_0) = (1 + \tau_t^k)\mu r_t^k = (1 + \tau_t^k)\mu(r_t + \delta).$$

The sequence of tax rates defined above ensure that firms optimize and markets clear for labor and capital. By definition of $\mathcal{A}_t$ and condition (i) of the lemma, the market for assets also clears given $\{r_t, T_t\}$. The final equilibrium condition involves government revenues and transfers. The total government revenue (before transfers) of this tax policy is given by

$$
\begin{aligned}
\text{Revenue} &= \tau_t^n w_0 N_0 + \tau_t^k r_t^k K_t + \tau_t^\pi \widetilde{\Pi}_t \\
&= (1 + \tau_t^n)w_0 N_0 + (1 + \tau_t^k)r_t^k K_t - (1 - \tau_t^\pi)\widetilde{\Pi}_t - w_0 N_0 - r_t^k K_t + \widehat{\Pi}_t \\
&= \frac{F_N(K_t, N_0)N_0 + F_K(K_t, N_0)K_t}{\mu} - \Pi_0 - w_0 N_0 - r_t^k K_t + \frac{(\mu - 1)F(K_t, N_0)}{\mu} \\
&= F(K_t, N_0) - \Pi_0 - w_0 N_0 - r_t^k K_t,
\end{aligned}
$$

where the third line uses $(1 - \tau_t^\pi)\widehat{\Pi}_t = \Pi_0$; the firm's first-order condition for labor and capital; and $\widehat{\Pi}_t = (\mu - 1)F/\mu$. The last line follows from Euler's theorem. Note that national income accounting implies

$$F(K_0, N_0) = \Pi_0 + w_0 N_0 + r_0^k K_0 + r_0 B_0.$$

Hence, we can replace $\Pi_0 + w_0 N_0 = F(K_0, N_0) - r_0^k K_0 - r_0 B_0$ and $r^k = r + \delta$ to obtain

$$\text{Revenue} = F(K_t, N_0) - F(K_0, N_0) - (r_t + \delta)K_t + (r_0 + \delta)K_0 + r_0 B_0. \tag{16}$$

As transfers equals revenue plus net debt issuance, we have

$$T_t \leq F(K_t, N_0) - F(K_0, N_0) - (r_t + \delta)K_t + (r_0 + \delta)K_0 + r_0 B_0 + B_{t+1} - (1 + r_t)B_t,$$

where the inequality allows for free disposal of government surpluses. This is condition (3), and thus ensures that the government has a non-negative surplus at every t given the proposed taxes, transfers, and debt issuances.   □

C.2 Proof of Corollary 1

Using

$$F(K_0, N_0) = w_0 N_0 + \Pi_0 + (r_0 + \delta)K_0 + r_0 B_0,$$

we have

$$C_t = F(K_0, N_0) - (r_0 + \delta)K_0 - r_0 B_0 + (1 + r_t)\mathcal{A}_t - \mathcal{A}_{t+1} + T_t.$$

Using $\mathcal{A}_t = K_t + B_t$, this is equivalent to

$$C_t + K_{t+1} = F(K_0, N_0) - (r_0 + \delta)K_0 - r_0 B_0 + (1 + r_t)(K_t + B_t) - B_{t+1} + T_t.$$

Substituting into (6) and re-arranging gives (3). □

C.3 Proof of Lemma 2

Proof. For a given v, let $T' = T^o + \widehat{T}v$ be the new transfer sequence. From the continuity condition, we have

$$C_t(r^o, T') - C^o \le |C_t(r^o, T') - C^o| \le Mv.$$

For $t = 0$, we have

$$\begin{aligned}
C_t(r^o, T') + K^\star &\le C^o + Mv + K^\star \\
&= F(K^o, N^o) - \delta K^o + K^\star + Mv \\
&= F(K^o, N^o) + (1 - \delta)K^o + (Mv + K^\star - K^o)
\end{aligned}$$

and hence the condition in Corollary 1 holds for $0 < v \le (K^o - K^\star)/M \equiv v_1$, as $K^o > K^\star$. For $t \ge 1$, it is sufficient if

$$Mv + C^o \le F(K^\star, N^o) - \delta K^\star,$$

or, using $C^o = F(K^o, N^o) - \delta K^o$,

$$Mv \le F(K^\star, N^o) - \delta K^\star - (F(K^o, N^o) - \delta K^o).$$

Letting $v_2 \equiv M^{-1}\left(F(K^\star, N^o) - \delta K^\star - (F(K^o, N^o) - \delta K^o)\right) > 0$, this condition is satisfied if $0 < v \le v_2$. Collecting, for $0 < v \le \min\{v_1, v_2\}$, the transfer scheme $T' = T^0 + \widehat{T}v$ is implementability and represents an RPI. □

C.4 Proof of Proposition 1

Proof. For a given v, let $r' \equiv r^o + \widehat{r}v$. Note that (r', T^o) is an RPI. Let us propose the following sequence of $\{K_t\}_{t=0}^\infty$:

$$K_t = K^o + R_k^{-1} \sum_{s=0}^\infty R_k^{-s}(C_{t+s}(r', T^o) - C^o) + hv, \quad \text{for } t \ge 1.$$

with $K_0 = K^o$. We will check that such sequence implements (r', T^o) for v small enough. Note that

$$\begin{aligned}
|K_t - K^o| &\le R_k^{-1} \sum_{s=0}^\infty R_k^{-s}|C_{t+s}(r', T^o) - C^o| + hv \\
&\le R_k^{-1} \sum_{s=0}^\infty R_k^{-s}vM + hv = \left[\left(\frac{1}{R_k - 1}\right)M + h\right]v \equiv M_0 v,
\end{aligned}$$

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where the second line uses property (ii). Then, there exists $v_1 < \epsilon$ such that $K_t > 0$ for all $t \geq 0$ and $v < v_1$. Let

$$\overline{F}_{KK} \equiv -\sup_K\{|F_{KK}(K, N^o)|: |K - K^0| \leq M_0 v_1\}.$$

As F_{KK} is continuous and this is a compact domain, $\overline{F}_{KK}$ is finite. Note that for $v < v_1$, Taylor's theorem implies that

$$F(K_t, N^0) + (1 - \delta)K_t = F(K^o, N^o) + (1 - \delta)K^o + R_k(K_t - K^o) + \frac{1}{2}F_{KK}(\widetilde{K}, N^o)(K_t - K^o)^2$$

for some $\widetilde{K}$ between K^o ad K_t. Using that $F(K^o, N^o) + (1 - \delta)K^o = C^o + K^o$ and that $|K_t - K^o| \leq M_0 v$, we have that

$$F(K_t, N^0) + (1 - \delta)K_t \geq C^o + K^o + R_k(K_t - K^o) + \frac{\overline{F}_{KK}}{2}(M_0 v)^2$$

Then, a sufficient condition for (6) from Corollary 1 is

$$C_0(r', T^o) + K_1 \leq C^o + K^o$$

$$C_t(r', T^o) + K_{t+1} \leq C^o + K^o + R_k(K_t - K^o) + \frac{\overline{F}_{KK}}{2}(M_0 v)^2, \text{ for all } t \geq 1.$$

For the first inequality, using the proposed K_1, we have that

$$\sum_{s=0}^{\infty} R_k^{-s}(C_s(r', T^o) - C^o) + hv \leq 0$$

which holds given (i).
For the second inequalities, using the proposed $\{K_t\}$, we have

$$\sum_{s=0}^{\infty} R_k^{-s}(C_{t+s}(r', T^o) - C^o) + hv \leq \sum_{s=0}^{\infty} R_k^{-s}(C_{t+s}(r', T^o) - C^o) + R_k hv + \frac{\overline{F}_{KK}}{2}(M_0 v)^2$$

$$0 \leq (R_k - 1)hv + \frac{\overline{F}_{KK}}{2}(M_0 v)^2$$

Given that $h > 0$, there exists $v_2 > 0$ such that

$$(R_k - 1)h \geq -\frac{\overline{F}_{KK}}{2}M_0^2 v$$

for all $v \in (0, v_2)$.
Let $\overline{v} = \min\{v_1, v_2\}$. Then (r', T^o) for any $v \in (0, \overline{v})$ is an implementable RPI. □

C.5 Proof of Corollary 2

Proof. Divide both sides of equation 12 by $R_K^{-\tau} A^o$, factor out R^o, and use the definition of $\xi_{t,\tau}$ to obtain (14). As shown in the text, this implies (11) is satisfied, which in turn is sufficient for (i) in Proposition 1. Condition (ii) holds by the differentiability of C_t, which is implied by the differentiability of $\mathcal{A}_t$ stated in the premise. □

C.6 Proof of Lemma 3

to be added

C.7 Proof of Proposition 2

Proof. Towards a contradiction, suppose there is an implementable RPI, (r, T). Given that we start from the laissez-faire allocation, this requires that T is non-negative. From the feasibility condition in Corollary 1, we have that there exists a sequence of K_t such that

$$w^o N^o + \Pi^o + (1 + r_t)\mathcal{A}(r, T) - \mathcal{A}_{t+1}(r, T) + T_t \le F(K_t, N^o) + (1 - \delta)K_t - K_{t+1}$$

$$\le \underbrace{F(K^o, N^o) + (1 - \delta)K^o}_{C^o + K^o} + R_k(K_t - K^o) - K_{t+1}$$

where the last inequality follows from concavity of F. Using that $R_k = 1 + r^o$ as $\mu = 1$, we have

$$w^o N^o + \Pi^o + T_t + (1 + r_t)\mathcal{A}_t(r, T) - \mathcal{A}_{t+1}(r, T) \le C^o + (1 + r^o)(K_t - K^o) - (K_{t+1} - K^o)$$

$$-r^o A^o + (1 + r_t)\mathcal{A}_t(r, T) - \mathcal{A}_{t+1}(r, T) + T_t \le (1 + r^o)(K_t - K^o) - (K_{t+1} - K^o)$$

And thus

$$\mathcal{A}_{t+1}(r, T) - K_{t+1} \ge (1 + r_t)(\mathcal{A}_t(r, T) - K_t) + (r_t - r^o)K_t + T_t$$

Note that starting from the laissez-faire implies that $K^o = A^o$, and thus $\mathcal{A}_{t+1}(r, T) - K_{t+1}$ is always non-negative, and turns strictly positive whenever $r_t > r^o$ or $T_t > 0$. Hence, we have that

$$\mathcal{A}_{t+1}(r, T) - K_{t+1} \ge (1 + r^o)(\mathcal{A}_t(r, T) - K_t) + (r_t - r^o)K_t + T_t$$

and given that $r^o > 0$ ($K^o < K^\star$ and $\mu = 1$), it follows that $\mathcal{A}_{t+1}(r, T) - K_{t+1}$ must necessarily go to infinity at t increases. The finite technology implies that K_t must remain bounded, and thus $\mathcal{A}_{t+1}(r, T) \to \infty$. The assumption in the proposition then implies that for any M there exists a s such that $C_s(r, T) > M$. For M sufficiently large, the resource constraint at s must be violated, generating the contradiction. $\square$

Appendix D   Computational Algorithm

This appendix describes the computational algorithm we use in solving the model. Our procedure consists of two steps. First, we compute the initial and final stationary equilibria. The initial one is the laissez-faire equilibrium and the final one has fiscal policy. We then compute the transition of this economy with shooting algorithms. We describe the algorithm for the dynamically efficient economy with markups.

D.1 Stationary Equilibrium

Initial   The laissez-faire initial stationary equilibrium is the standard Bewley-Hugget-Aiyagari model. We compute it with a value function iteration over a savings grid and solve for the equilibrium wages and interest rates that clear markets. The objects we record are prices $\{w_0, r_0\}$, aggregate capital, labor, profits, and the limiting distribution of households over idiosyncratic state $\{a, z\}$—namely $\{K_0, N_0, \Pi_0, \Lambda_0(a, z)\}$. The objects in the initial stationary equilibrium are denoted by 0.

Final   The final stationary equilibrium is indexed by H.

1. We set a target levels for capital K_H and debt B_H.

2. Choose fiscal policies $\{\tau_H^n, \tau_H^\pi\}$ to keep wages and profits as in the initial equilibrium using

$$F_L(K_H, N_0) = \mu(1 + \tau_H^n)w_0$$
$$\Pi_0 = (1 - \tau_H^\pi)(\mu - 1)F(K_H, N_0)/\mu.$$

3. Guess an equilibrium interest rate r_H.

4. We recover the initial implied τ_H^k from

$$F_K(K_H, N_0) = \mu(1 + \tau_H^k)(r_H + \delta).$$

5. Use the government budget as a constraint to recover transfers T_H given our settings and the guess for interest rates:

$$T_H = F(K_H, N_0) - F(K_0, N_0) - (r_H + \delta)K_H + (r_0 + \delta)K_0 + r_H B_H$$

6. Solve household's problem:

$$V_H(a, z; r_H) = \max_{a' \geq \underline{a}, c, n} \phi(x(c, n), EV_H(a', z'; r_H))$$

$$\text{subject to: } c + a' \leq w_0 zn + (1 + r_H)a + T_H.$$

- Gives value and savings policy functions $V_H(a, z; r_H)$, $a_H'(a, z; r_H)$ and labor supply without wealth effects $n_H(z)$ and a household distribution $\Lambda_H(a, z; r_H)$.

7. Use the asset market clearing condition and firm's optimal capital condition to obtain a new guess on interest rates $\tilde{r}_H$:

$$B_H + \tilde{K}_H = \int a_H'(a, z; r_H)\pi(z', z)d\Lambda_H(a, z; r_H)$$

$$F_K(\tilde{K}_H, N_0) = \mu(1 + \tau_H^k)(\tilde{r}_H + \delta)$$

8. We go back to step 3 and repeat the procedure until $\tilde{K}_H$ is close to the target capital level.

D.2   Transition

At time 0, the government announces a sequence of fiscal policies that implements a sequence of capital and debt $\{K_t, B_t\}_{t=0}^H$. We will assume that at period H, the economy is in the final stationary equilibrium.

1. Choose fiscal policies $\{\tau_t^n, \tau_t^\pi\}$ to keep wages and profits as in the initial equilibrium using

$$F_L(K_t, N_0) = \mu(1 + \tau_t^n)w_0$$
$$\Pi_0 = (1 - \tau_t^\pi)(\mu - 1)F(K_t, N_0)/\mu.$$

2. Guess sequence of interest rates $\{r_t\}_{t=0}^H$.

- Recover the capital taxes given our target capital sequence and interest rate guess:

$$F_K(K_t, N_0) = \mu(1 + \tau_t^k)(r_t + \delta).$$

- Recover transfers from government budget constraint using sequence of debt

$$T_t = F(K_t, N_0) - F(K_0, N_0) - (r_t + \delta)K_t + (r_0 + \delta)K_0 + B_{t+1} - (1 + r_t)B_t$$

3. Solve household's problem backwards.

- Start with period $H - 1$ problem. Note that we have the value at period H from the stationary equilibrium:

$$V_t(a, z) = \max_{a' \geq \underline{a}, c, n} \phi(x(c, n), h(V_{t+1}(a', z')))$$

$$\text{subject to: } c + a' \leq w_0 z n + (1 + r_t)a + T_t$$

- Store saving decision rules: $a_t'(a, z)$.

4. Iterate forwards, update interest rates: The resulting aggregate savings from Step 2 will not be equal to the targets.

- Start with initial distribution $\Lambda_0(a, z)$ and apply the decision rules from above.
- Use the asset market clearing condition to obtain the resulting capital sequence $\tilde{K}_{t+1}$:

$$B_{t+1} + \tilde{K}_{t+1} = \int a_t'(a, z)\pi(z', z)d\Lambda_t(a, z)$$

- Use firm's optimal capital condition to obtain a candidate new sequence of interest rates $\tilde{r}_t$:

$$F_K(\tilde{K}_t, N_0) = \mu(1 + \tau_t^k)(\tilde{r}_t + \delta).$$

- Update the sequence of interest rates so that $r_t^{new} = \lambda r_t^{old} + (1 - \lambda)\tilde{r}_t$ for attenuation parameter $\lambda = 0.5$. If the new sequence of interest rates is close enough to the old sequence, we stop; otherwise, we go back to step 2.

Appendix E The Growth Economy

In this appendix, we show how the key expressions of Section 2 are modified by the presence of exogenous labor-augmenting technological growth. The derivations are standard and are included for completeness.

Assume technology is given by

$$Y_t = F(K_t, (1 + g)^t L_t),$$

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where $g \geq 0$ is the constant rate of growth of labor-augmenting technology. Letting a tilde denote variables divided by $(1 + g)^t$, constant returns implies

$$\tilde{Y}_t \equiv (1 + g)^{-t} Y_t = F(\tilde{K}_t, L_t).$$

The representative firm's first-order conditions are (dropping t subscripts)

$$F_k(\tilde{K}, L) = \mu(1 + \tau^k) r^k$$
$$F_l(\tilde{K}, L) = \mu(1 + \tau^n) \tilde{w}.$$

We also have $\widetilde{\Pi} = (1 - \tau^\pi)(\mu - 1) F(\tilde{K}, L) / \mu$.

Given the absence of a wealth effect on labor supply, we assume that the disutility of working grows at rate g as well (dropping i and t indicators):

$$x(c, n) = c - (1 + g)^t v(n),$$

giving us

$$\tilde{x}(\tilde{c}, n) \equiv (1 + g)^{-t} x(c, n) = \tilde{c} - v(n).$$

We also assume that the borrowing constraint is scaled by $(1 + g)^t$.

We can write the household's problem as

$$V_t(a, z, \theta) = \max_{a', n, c} \phi(x(c, n), h(V_{t+1}(a', z', \theta')))$$

$$\text{s.t. } c + a' \leq w_t z n + \theta \Pi_t + (1 + r_t) a + T_t$$

$$a' \geq (1 + g)^{t+1} \underline{a},$$

where we have altered the last constraint to account for growth and h is a certainty equivalent operator. The constraint set can be rewritten as

$$\tilde{c} + (1 + g) \tilde{a}' \leq \tilde{w}_t z n + \theta \widetilde{\Pi}_t + (1 + r_t) \tilde{a} + \tilde{T}_t$$

$$\tilde{a}' \geq \underline{a}.$$

Thus, if (c, n, a') is feasible at time t, then $(\tilde{c}, n, \tilde{a}')$ satisfies the normalized constraint set and vice versa. If we assume ϕ is constant-returns in x and h is homogeneous of degree 1, if $V_t(a, z, \theta)$ satisfies the consumer's Bellman equation, then $\tilde{V}_t(\tilde{a}, z, \theta) \equiv (1 + g)^{-t} V_t(a, z, \theta)$ satisfies

$$\tilde{V}_t(\tilde{a}, z, \theta) = \max_{\tilde{c}, n, \tilde{a}'} \phi(\tilde{x}(\tilde{c}, n), (1 + g) h(\tilde{V}_{t+1}(\tilde{a}', z', \theta'))),$$

subject to the normalized constraint set, and vice versa.[41]

Note that for an interior optimum for n, the first-order condition can be expressed as follows:

$$v'(n) = z \tilde{w}.$$

---

[41] For the simulations, we use $\phi(x, h) = ((1 - \beta) x^{1-\zeta} + \beta h^{1-\zeta})^{1/(1-\zeta)}$. In this case, we can define $\tilde{\beta} \equiv \beta(1 + g)^{1-\zeta}$ and write $\tilde{\phi}(\tilde{x}, h) = ((1 - \tilde{\beta}) \chi \tilde{x}^{1-\zeta} + \tilde{\beta} h^{1-\zeta})^{1/(1-\zeta)}$, where $\chi \equiv (1 - \beta)/(1 - \tilde{\beta})$. This is well defined as long as $\tilde{\beta} \leq 1$. Growth can be accommodated by re-scaling the discount factor, as expected with homogeneous preferences.

Hence, labor supply is constant as long as $\tilde{w}$ remains constant.

The government's budget constraint can be rewritten in normalized form:

$$\tilde{T}_t = \tau_t^n \tilde{w}_t N_t + \tau_t^k r_t^k \tilde{K}_t + \tau_t^\pi \widetilde{\Pi}_t / (1 - \tau_t^\pi) + (1 + g)\tilde{B}_{t+1} - (1 + r_t)\tilde{B}_t.$$

Let $\tilde{X}_t \equiv \tau_t^n \tilde{w}_0 N_0 + \tau_t^k r_t^k \tilde{K}_t + \tau_t^\pi \widetilde{\Pi}_0 / (1 - \tau_t^\pi)$ denote normalized tax revenue before transfers when keeping after tax normalized wages and profits constant. Following the same steps as the proof of Lemma 1, we have

$$\tilde{X}_t = F(\tilde{K}_t, N_0) - F(\tilde{K}_0, N_0) - (r_t + \delta)\tilde{K}_t + (r_0 + \delta)\tilde{K}_0.$$

Condition (iii) of Lemma 1 (equation (3)) becomes

$$(1 + g)\tilde{B}_{t+1} - (1 + r_t)\tilde{B}_t - \tilde{T}_t \geq F(\tilde{K}_0, N_0) - F(\tilde{K}_t, N_0) - (r_0 + \delta)\tilde{K}_0 + (r_t + \delta)\tilde{K}_t.$$

Condition (ii) becomes $\tilde{T}_t \geq -(r_t - r_0)\underline{\tilde{a}}$, and condition (i) remains unchanged. Note that in a steady state (that is, relevant aggregates grow at rate g), Condition (iii) becomes

$$(g - r_{ss})\tilde{B}_{ss} - \tilde{T}_{ss} \geq F(\tilde{K}_0, N_0) - F(\tilde{K}_{ss}, N_0) - (r_0 + \delta)\tilde{K}_0 + (r_t + \delta)\tilde{K}_{ss}.$$

Hence, debt increases government revenues in the steady state as long as $g > r_{ss}$. Expressions in Claims 1 and 2 are adjusted in a similar fashion to obtain normalized equivalents.

Appendix F   Additional Comparative Statics with Respect to Preference Parameters

Figure A.2: Alternative Relative Risk Aversion Coefficients
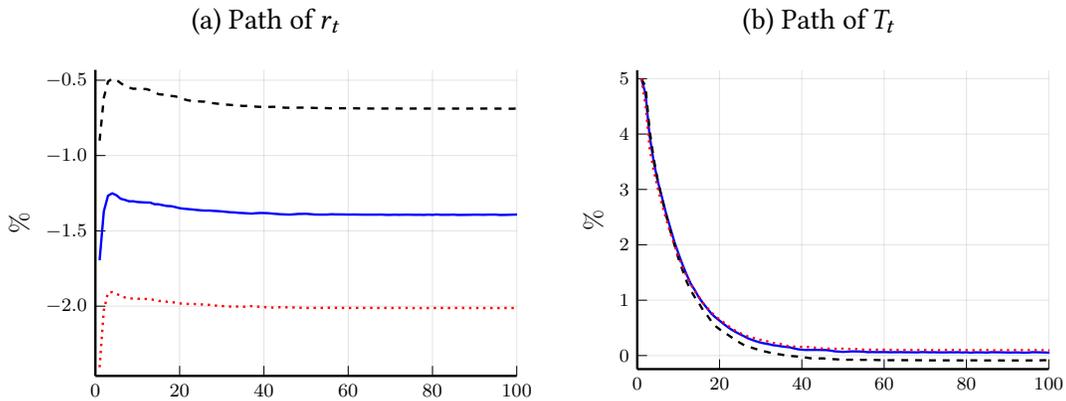
(a) Path of r_t (b) Path of T_t

Note: This figure displays the path of interest rates (Panel a) and transfers as a percentage of laissez-faire output (Panel b) associated with the baseline fiscal policy under alternative preference parameterizations for the coefficient of relative risk aversion (CRRA). In both panels, the solid line is the benchmark CRRA=5.5; the dotted red line is CRRA=10.0; and the dashed black line is CRRA=2.0.

Figure A.3: Alternative Discount Factors
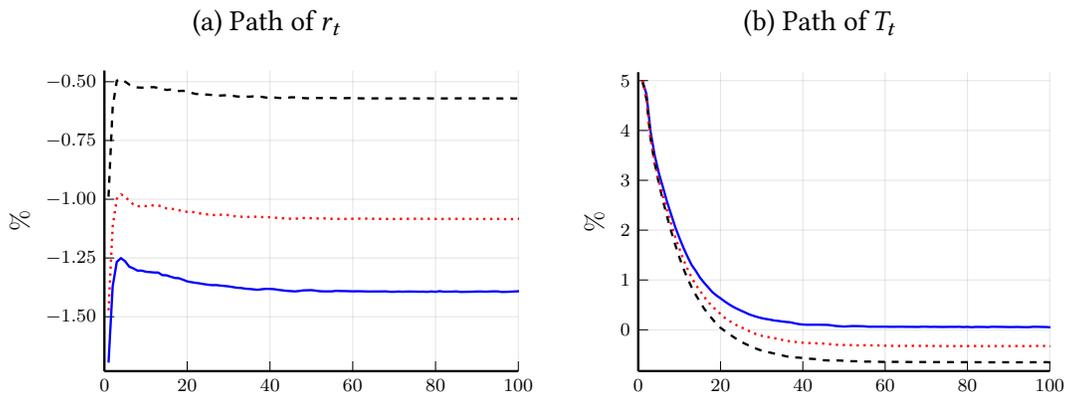
(a) Path of r_t (b) Path of T_t

Note: This figure displays the path of interest rates (Panel a) and transfers as a percentage of laissez-faire output (Panel b) associated with the baseline fiscal policy under alternative preference parameterizations for time discount factors. In both panels, the solid line is the benchmark β-0.993; the dotted red line is β=0.98; and the dashed black line is β=0.97.